SECTION 1 – 1 : SETS AND REAL NUMBERS المجموعات و الأعداد الحقيقية

بعض التعريفات المهمة:

Set : المجموعة

A set is a collection of elements المجموعة عبارة عن مجموعة من العناصر

For Example : Set $B = \{a, b, 1, 4\}$

Subset : مجموعة جزئية

If every element of set A is also an element of set B . then A is subset of B ($A \subseteq B$)

اذا كان كل عنصر من مجموعة A يكون عنصر في B فان A مجموعة جزئية من B

Equality of Sets : المجموعات المتساوية

If the sets A and B contain the same elements then $A = B$

اذا كانت المجموعات تحتوي نفس العناصر فانهما متساويان او متطابقان

Union of two sets : اتحاد مجموعتان

The set that contains all elements in A or in B or in both A and B

اتحاد مجموعتين يعطي مجموعة تعطي جميع العناصر في A و جميع العناصر B او كليهما (كل العناصر)

$A \cup B = \{x : x \in A \text{ or } x \in B\}$

Intersection of two sets : تقاطع مجموعتين

The set that consists of all elements in A and B at the same time

تقاطع مجموعتين يعطي مجموعة تحتوي العناصر المشتركة بين المجموعتين

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

The difference of two sets : فرق بين مجموعتين

The set of all elements in A but not in B

فرق بين المجموعتين تعطي مجموعة العناصر الموجودة بالمجموعة A و ليست موجودة بالمجموعة B

$A - B = \{x : x \in A \text{ and } x \notin B\}$

Ex : list all elements in the following sets using set notation

1. Vowels in the English alphabet. الحروف الهجائية المتحركة بالإنجليزية
2. First seven prime numbers. أول سبع ارقام أولية
3. Even integrs between 50 and 63 الاعداد الصحيحة الزوجية بين 50 و 63

solution:

1. Vowels in the English alphabet

$V = \{a , e , i , o, u\}$

2. First seven prime numbers.

$P = \{2 , 3 , 5 , 7 , 11\}$

3. Even integrs between 50 and 63

$E = \{52 , 54 , 56 , 58 , 60 , 62\}$

Ex : Identify the elements in each set , assuming

$A = \{w , x , y , z\}$, $B = \{x , y\}$, $C = \{x , y , z\}$, and $D = \{z\}$

1. $A \cup B = \{w , x , y , z\}$, 2. $A \cap B = \{x , y\}$, 3. $B \cap C = \{x , y\}$
4. $B \cap D = \varnothing$ 5. $B \cup D = \{x , y , z\}$
6. $B \cap (C \cup D) = \{x , y\} \cap \{x , y , z\} = \{x , y\}$
7. $(A \cap C) \cup D = \{x , y , z\} \cup \{z\} = \{x , y , z\}$
8. $B \cup \varnothing = \{x , y\}$ 9. $C \cap \varnothing = \varnothing$
10. $A - B = \{w , z\}$
11. $B - C = \phi$

The sets of numbers

* Natural numbers : $\mathbb{N} = \{1, 2, 3, ...\}$ الأعداد الطبيعية

* Integers numbers $\mathbb{Z} = \{......., -3, -2, -1, 0, 1, 2, 3, ...\}$ الأعداد الصحيحة

$\mathbb{Z}^+ = \{1, 2.3, ...\}$

$\mathbb{Z}^- = \{-1. -2, -3, ...\}$

$\mathbb{Z} = \mathbb{Z}^- \cup \{0\} \cup \mathbb{Z}^+$

* Rational numbers $\mathbb{Q} = \left\{ \frac{p}{q} : p.q \in \mathbb{Z} : q \neq 0 \right\}$ الأعداد الكسرية

For example : $\frac{2}{5}$, $\frac{3}{7}$, 5 , $-\frac{3}{10}$, $0.\overline{34}$

* Irrational numbers I الأعداد الغير منطقية

For example : $\sqrt{2}$, π , e

* Real numbers $\mathbb{R}$ الأعداد الحقيقية

$\mathbb{R} = \mathbb{Q} \cup I$

- ملحوظة : الأعداد الحقيقية تحتوي جميع الأعداد السابقة و لا تحتوي جذور الأعداد السالبة $\sqrt{-4} \notin \mathbb{R}$

** $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

Ex : Answer by **True** or **False**

1. $0 \in \mathbb{N}$	(False)	2. $\mathbb{Q} \subset \mathbb{R}$	(True)
3. $I \in R$	(False)	4. $\mathbb{Q} \subseteq \mathbb{Z}$	(False)
5. $\{2,2,2\} = \{2\}$	(True)	6. $\{1,2,3\} = \{3,1,2\}$	(True)
7. $1 = \{1\}$	(False)	8. $1 \subseteq \{1,2\}$	(False)
9. $\{1\} \in \{1,2\}$	(False)	10. $\{1\} \subseteq \{1,2\}$	(True)

ملحوظة هامة :

- العلامة $\subset$ or $\subseteq$ مجموعة جزئية تستخدم في العلاقة بين المجموعات
- العلاقة $\in$ علاقة الانتماء تستخدم في علاقة انتماء عنصر داخل مجموعة

23. Put a check mark in each box if the number is an element of that set.

	Natural	Integer	Rational	Irrational	Real
2	✓	✓	✓		✓
$\frac{3}{5}$			✓		✓
$\sqrt{10}$				✓	✓
$\sqrt{2}$				✓	✓
0.35			✓		✓
0		✓	✓		✓

$\frac{6}{0}$	Not	Not	Not	Not	Not
-2		✓	✓		✓
0.25481931...				✓	✓
0.262626...			✓		✓
$\frac{0}{0}$	Not	Not	Not	Not	Not
$\sqrt{16}=4$	✓	✓	✓		✓
$\sqrt{-1}$	Not	Not	Not	Not	Not

<u>Properties of the Fractions</u>

نعطي الخاصية ومثال عليها للتوضيح

Property	Example
* $\frac{a}{b}=\frac{a}{b}\cdot\frac{c}{c}=\frac{ac}{bc}$ $b\neq 0, c\neq 0$	* $\frac{3}{5}=\frac{3}{5}\cdot\frac{2}{2}=\frac{6}{10}$
* If $\frac{a}{b}=\frac{c}{d}$ then $a\cdot d=c\cdot b$ $b\neq 0, c\neq 0$	* $\frac{3}{4}=\frac{6}{8}$ then $(3)(8)=(6)(4)$
* $\frac{a}{c}\pm\frac{b}{c}=\frac{a\pm b}{c}$ $c\neq 0$	* $\frac{4}{5}+\frac{3}{5}=\frac{4+3}{5}=\frac{7}{5}$
* $\frac{a}{b}\pm\frac{c}{d}=\frac{ad\pm cb}{bd}$ $b\neq 0$, $d\neq 0$	* $\frac{4}{5}+\frac{3}{7}=\frac{4\cdot 7+3\cdot 5}{5\cdot 7}=\frac{43}{35}$
* $\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{db}$ $b\neq 0$, $d\neq 0$	* $\frac{3}{7}\cdot\frac{2}{5}=\frac{3\cdot 2}{7\cdot 5}=\frac{6}{35}$
* $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}$ $b\neq 0$, $c\neq 0$, $d\neq 0$	* $\frac{5}{7}\div\frac{4}{3}=\frac{5}{7}\cdot\frac{3}{4}=\frac{15}{28}$

Examples , Perform the indicated operations

نفذ العمليات المعطاة

1. $\frac{4}{7}+\frac{2}{5}$

solution: نوحد المقامات ثم نجمع

$$\frac{4\cdot5}{7\cdot5}+\frac{2\cdot7}{5\cdot7}=\frac{20}{35}+\frac{14}{35}=\frac{34}{35}$$

2. $2\cdot\frac{1}{3}-\frac{3}{5}$

solution:

$$*\ 2\cdot\frac{1}{3}-\frac{3}{5}=\frac{2}{1}\cdot\frac{1}{3}-\frac{3}{5}$$
$$=\frac{2}{3}-\frac{3}{5}$$
$$=\frac{2\cdot5}{3\cdot5}-\frac{3\cdot3}{5\cdot3}$$
$$=\frac{10}{15}-\frac{9}{15}=\frac{1}{15}$$

3. $\dfrac{\frac{4}{11}}{\frac{7}{33}}$

solution:

$$*\ \frac{4}{11}\div\frac{7}{33}=\frac{4}{11}\cdot\frac{33}{7}$$
$$=\frac{132}{77}=\frac{12}{7}$$

4. $[(8+7)\div5]\cdot2-9$

solution:

$$[(8+7)\div5]\cdot2-9=[15\div5]\cdot2-9$$ تبسيط ما داخل الأقواس أولا
$$=[3]\cdot2-9$$
$$=6-9=-3$$

5. $-6\frac{1}{4}\cdot\frac{3}{5}$

solution:

$$6\frac{1}{4} = 6 + \frac{1}{4} = \frac{6\cdot 4}{1\cdot 4} + \frac{1}{4} = \frac{24}{4} + \frac{1}{4} = \frac{25}{4}$$

$$-6\frac{1}{4}\cdot\frac{3}{5} = -\frac{25}{4}\cdot\frac{3}{5} = -\frac{75}{20} = -\frac{75\div 5}{20\div 5} = -\frac{15}{4}$$

6. $5[(6+3\cdot 2)-2(8-5)]$

solution:

$$*\ 5[(6+3\cdot 2)-2(8-5)] = 5[(6+6)-2(3)] = 5[12-6] = 5[6] = 30$$

7. $-(41-7\cdot 4)+30\div[6-(-4)]-12$

solution:

$$*\ -(41-7\cdot 4)+30\div[6-(-4)]-12 = -(41-28)+30\div[6+4]-12 = -(13)+30\div 10-12 = -13+3-12 = -22$$

Exponents Properties خواص الأسس

Let n , $m \in \mathbb{Z}^+$ and a , $b \in \mathbb{R}$

Property	Example
* $a^n = a \cdot a \cdot a \dots a$ (n times)	* $4^3 = 4 \cdot 4 \cdot 4$
* $a^n \cdot a^m = a^{n+m}$	* $2^3 \cdot 2^4 = 2^{3+4} = 2^7$
* $\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$	* $\frac{4^{11}}{4^3} = 4^{11-3} = 4^8$
* $\left(a^n\right)^m = a^{n \cdot m}$	* $\left(3^2\right)^5 = 3^{2 \cdot 5} = 3^{10}$
* $(a \cdot b)^n = a^n \cdot b^n$	* $(3 \cdot 4)^5 = 3^5 \cdot 4^5$
* $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, $b \neq 0$	* $\left(\frac{3}{5}\right)^6 = \frac{3^6}{5^6}$
* $a^{-n} = \frac{1}{a^n}$, $a \neq 0$	* $(5)^{-3} = \frac{1}{5^3}$
* $\left(a^n b^m\right)^p = \left(a^n\right)^p \cdot \left(b^m\right)^p = a^{np} \cdot b^p$	* $\left(5^2 \cdot 6^3\right)^4 = \left(5^2\right)^4 \cdot \left(6^3\right)^4 = 5^{2 \cdot 4} \cdot 6^{3 \cdot 4} = 5^8 \cdot 6^{12}$
* $\left(\frac{a^n}{b^m}\right)^p = \frac{\left(a^n\right)^p}{\left(b^m\right)^p} = \frac{a^{np}}{b^{mp}}$	* $\left(\frac{2^3}{5^4}\right)^2 = \frac{\left(2^3\right)^2}{\left(5^4\right)^2} = \frac{2^6}{5^8}$

Examples: Simplify and express answers using positive exponents only

1. $x^5 x^{-2}$

solution:

* $x^5 x^{-2} = x^{5-2} = x^3$

2. $y^{1/5} \cdot y^{4/5}$

solution:

* $y^{1/5} \cdot y^{4/5} = y^{\frac{1}{5}+\frac{4}{5}} = y$

3. $(6x^3)(4x^7)(x^{-5})$

solution:

* $(6 \cdot 4)x^3 x^7 x^{-5} = 24x^{3+7-5} = 24x^5$

4. $(2y)(3y^2)(5y^{-4})$

solution:

* $(2y)(3y^2)(5y^{-4}) = (2 \cdot 3 \cdot 5)(y \cdot y^2 \cdot y^{-4})$

$= 30y^{1+2-4}$

$= 30y^{-1} = \dfrac{30}{y}$

5. $\dfrac{15x^{-4}y^3}{18x^{-3}y^{-5}}$

solution:

* $\dfrac{15x^{-4}y^3}{18x^{-3}y^{-5}} = \dfrac{15}{18} \cdot \dfrac{x^{-4}}{x^{-3}} \cdot \dfrac{y^3}{y^{-5}}$

$= \dfrac{15}{18} \cdot x^{-4-(-3)} \cdot y^{3-(-5)}$

$= \dfrac{5}{6} x^{-1} y^8 \quad = \dfrac{5y^8}{6x}$

6. $(49a^4 b^{-2})^{1/2}$

solution:

* $(49a^4 b^{-2})^{1/2} = (49)^{1/2}(a^4)^{1/2}(b^{-2})^{1/2}$

$= 7a^2 b^{-1} = \dfrac{7a^2}{b}$

7. $\left(\frac{x^{4}y^{-1}}{x^{-2}y^{3}}\right)^{2}$

solution:

$$* \left(\frac{x^{4}y^{-1}}{x^{-2}y^{3}}\right)^{2} = \left(x^{4-(-2)}\cdot y^{-1-3}\right)^{2}$$

$$= \left(x^{6}y^{-4}\right)^{2}$$

$$= (x^{6})^{2}(y^{-4})^{2}$$

$$= x^{12}y^{-8} = \frac{x^{12}}{y^{8}}$$

8. $\left(\frac{4x^{-2}}{y^{4}}\right)^{-1/2}$

solution:

$$* \left(\frac{4x^{-2}}{y^{4}}\right)^{-1/2} = \frac{(2^{2})^{-1/2}(x^{-2})^{-1/2}}{(y^{4})^{-1/2}}$$

$$= \frac{2^{-1}x}{y^{-2}} = \frac{xy^{2}}{2}$$

9. $\left(\frac{25x^{5}y^{-1}}{16x^{-3}y^{-5}}\right)^{1/2}$

solution:

$$* \left(\frac{25x^{5}y^{-1}}{16x^{-3}y^{-5}}\right)^{1/2} = \left(\frac{25}{16}\cdot x^{5-(-3)}\cdot y^{-1-(-5)}\right)^{1/2}$$

$$= \left(\frac{25}{16}x^{8}y^{4}\right)^{1/2}$$

$$= \frac{25^{1/2}}{16^{1/2}}(x^{8})^{1/2}(y^{4})^{1/2}$$

$$= \frac{5}{4}x^{4}y^{2}$$

10. $-3(x^2+3)^{-4}(3x^2)$

solution:

$$*\quad -3(x^2+3)^{-4}(3x^2)=\frac{-3(3x^2)}{(x^2+3)^4}$$

$$=\frac{-9x^2}{(x^2+3)^4}$$

11. $\frac{4x^4y}{3x^3y^3}\cdot\frac{7x^3y^5}{14x^9y^2}$

solution:

$$\frac{4x^4y}{3x^3y^3}\cdot\frac{7x^3y^5}{14x^9y^2}=\frac{4}{3}x^{4-3}y^{1-3}\cdot\frac{7}{14}x^{3-9}y^{5-2}$$

$$=\frac{4}{3}xy^{-2}\cdot\frac{1}{2}x^{-6}y^{3}$$

$$=\frac{2}{3}x^{-5}y$$

$$=\frac{2y}{3x^5}$$

12. $\frac{3^n\cdot 9^{n-1}\cdot 27^{3n-2}}{81^{2n-1}}$

solution:

$$*\quad \frac{3^n\cdot 9^{n-1}\cdot 27^{3n-2}}{81^{2n-1}}=\frac{3^n\cdot(3^2)^{n-1}\cdot(3^3)^{3n-2}}{(3^4)^{2n-1}}$$

$$=\frac{3^n\cdot 3^{2(n-1)}\cdot 3^{3(3n-2)}}{3^{4(2n-1)}}$$

$$=\frac{3^n\cdot 3^{2n-2}\cdot 3^{9n-6}}{3^{8n-4}}$$

$$=\frac{3^{n+2n-2+9n-6}}{3^{8n-4}}$$

$$=\frac{3^{12n-8}}{3^{8n-4}}$$

$$=3^{12n-8-(8n-4)}$$

$$=3^{4n-4}$$

Properties of Roots

Let $n > 1$, $n \in \mathbb{Z}^+$ and x , y $\in [0,\infty)$

Property	Example
* $\sqrt[n]{x^n} = x$	* $\sqrt[5]{x^5} = x$
* $\sqrt[n]{xy} = \sqrt[n]{x} \cdot \sqrt[n]{y}$	* $\sqrt[4]{3 \cdot 6} = \sqrt[4]{3} \cdot \sqrt[4]{6}$
* $\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$	* $\sqrt{\frac{5}{11}} = \frac{\sqrt{5}}{\sqrt{11}}$

ملحوظات هامة:

1. $\sqrt[n]{x} = x^{1/n}$, for $n \in \mathbb{Z}^+$

   For example : $\sqrt[3]{5} = 5^{1/3}$

2. الجذر الزوجي للعدد السالب لا يعطي عدد حقيقي فمثلا $\sqrt{-16} \in \mathbb{R}$

3. الجذر الفردي مقبول للعدد الموجب و السالب فمثلا $\sqrt[3]{-27} = -3$ و $\sqrt[3]{27} = 3$

Ex : Simplify

1. $\sqrt[7]{\left(6x^3y^4\right)^7}$

solution:

* $\sqrt[7]{\left(6x^3y^4\right)^7} = \left[\left(6x^3y^4\right)^7\right]^{\frac{1}{7}}$

$= 6x^3y^4$

2. $\sqrt{6}\sqrt{8}$

solution:

* $$\sqrt{6}\sqrt{8}=\sqrt{48}=\sqrt{3\cdot 16}$$
$$=\sqrt{3}\cdot\sqrt{16}$$
$$=4\sqrt{3}$$

3. $\sqrt[5]{\dfrac{x^{10}}{32}}$

solution:

* $$\sqrt[5]{\frac{x^{10}}{32}}=\frac{\sqrt[5]{x^{10}}}{\sqrt[5]{32}}$$
$$=\frac{(x^{10})^{1/5}}{(2^5)^{1/5}}=\frac{x^2}{2}$$

4. $\sqrt[3]{2x^2y^4}\cdot\sqrt[3]{4x^5y}$

solution:

* $$\sqrt[3]{2x^2y^4}\cdot\sqrt[3]{4x^5y}=\sqrt[3]{2x^2y^4\cdot 4x^5y}$$
$$=\sqrt[3]{8x^7y^5}$$
$$=\sqrt[3]{2^3x^6y^3\cdot xy^2}$$
$$=2x^2y\cdot\sqrt[3]{xy^2}$$

5. $\sqrt[3]{9x^2y^3}\cdot\sqrt[3]{6x^8y^2}$

solution:

* $$\sqrt[3]{9x^2y^3}\cdot\sqrt[3]{6x^8y^2}=\sqrt[3]{9x^2y^3\cdot 6x^8y^2}$$
$$=\sqrt[3]{54x^{10}y^5}$$
$$=\sqrt[3]{(27\cdot 2)(x^9\cdot x)(y^3\cdot y)}$$
$$=\sqrt[3]{27x^9y^3\cdot 2xy}$$
$$=3x^3y\ \sqrt[3]{2xy}$$

6. $(-64)^{2/3}$

solution:

* $(-64)^{2/3} = \left((-4)^3\right)^{2/3}$

$= (-4)^2 = 16$

7. $(3y)^{1/3}\left(4y^{2/7}\right)$

solution:

* $(3y)^{1/3}\left(4y^{2/7}\right) = 3^{1/3}y^{1/3} \cdot 4y^{2/7}$

$= 4 \cdot 3^{1/3}\, y^{\frac{1}{3}+\frac{2}{7}}$

$= 4 \cdot 3^{1/3}\, y^{\frac{13}{21}}$

8. $\left(\dfrac{27x^{1/3}}{x^{4/5}}\right)^{1/3}$

solution:

* $\left(\dfrac{27x^{1/3}}{x^{4/5}}\right)^{1/3} = \left(27x^{\frac{1}{3}-\frac{4}{5}}\right)^{1/3}$

$= \left(27x^{\frac{-7}{15}}\right)^{1/3}$

$= 27^{\frac{1}{3}} \cdot x^{-\frac{7}{45}}$

$= \dfrac{3}{x^{7/45}}$

9. $\left(x^{1/2}+2y^{1/2}\right)\left(3x^{1/2}-y^{1/2}\right)$

solution:

* $\left(x^{1/2}+2y^{1/2}\right)\left(3x^{1/2}-y^{1/2}\right) = 3x - x^{1/2}y^{1/2} + 6y^{1/2}x^{1/2} - 2y$

$= 3x + 5x^{1/2}y^{1/2} - 2y$

10. $\left(3x^{1/5}y^{-3/5}\right)^5$

solution:

* $\left(3x^{1/5}y^{-3/5}\right)^5 = 3^5 \cdot \left(x^{1/5}\right)^5 \cdot \left(y^{-3/5}\right)^5$

$= 243 \cdot x \cdot y^{-3}$

$= \dfrac{243x}{y^3}$

11. $x \cdot \sqrt[5]{3^6 x^7 y^{11}}$

solution:

* $x \cdot \sqrt[5]{3^6 x^7 y^{11}} = x \cdot \sqrt[5]{3^5 x^5 y^{10} \cdot 3x^2 y}$

$= x \cdot 3xy^2 \sqrt[5]{3x^2 y}$

$= 3x^2 y^2 \sqrt[5]{3x^2 y}$

12. $\dfrac{\sqrt[5]{32u^{12}v^8}}{u \cdot v}$

solution:

* $\dfrac{\sqrt[5]{32u^{12}v^8}}{u \cdot v} = \dfrac{\sqrt[5]{2^5 u^{10} v^5 \cdot u^2 v^3}}{u \cdot v}$

$= \dfrac{2u^2 v \cdot \sqrt[5]{u^2 v^3}}{u \cdot v}$

$= 2u \cdot \sqrt[5]{u^2 v^3}$

Section (1 – 3) : RATIONAL EXPRESIONS العبارات الكسرية

Basic Identities متطابقات اساسية

Let $a, b \in \mathbb{R}$

1. $a^2 - b^2 = (a+b)(a-b)$
2. $(a \pm b)^2 = a^2 \pm 2ab + b^2$
3. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
4. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

- الأسئلة دمج من افكار related و exercise لتحتوي جميع أفكار المنهج

Example : Reduce each expression to the simplest form اختزل كل تعبير الى أبسط صورة

1. $\dfrac{x^2 - 8x + 16}{x^2 - 16}$, $x \neq \pm 2$

solution:

$$* \ \frac{x^2 - 8x + 16}{x^2 - 16} = \frac{(x-4)(x-4)}{(x-4)(x+4)}$$
$$= \frac{x-4}{x+4}$$

Related Problem (1)

2. $\dfrac{x^3 - y^3}{x^2 - xy}$, $x \neq y$, $x \neq 0$

solution:

$$* \ \frac{x^3 - y^3}{x^2 - xy} = \frac{(x-y)(x^2 + xy + y^2)}{x(x-y)}$$
$$= \frac{x^2 + xy + y^2}{x}$$

Exercise 2

Exercice 2. $\frac{b^2}{2a} \div \left(\frac{b^2}{a^2} \cdot \frac{a}{3b}\right)$; $a \neq 0, b \neq 0$

solution:

* $\frac{b^2}{2a} \div \left(\frac{b^2}{a^2} \cdot \frac{a}{3b}\right) = \frac{b^2}{2a} \div \left(\frac{b}{a} \cdot \frac{1}{3}\right)$ نختزل ما داخل القوس أولا

$= \frac{b^2}{2a} \div \frac{b}{3a}$ نحول القسمة الى ضرب و نقلب الكسر

$= \frac{b^2}{2a} \cdot \frac{3a}{b}$

$= \frac{b}{2} \cdot \frac{3}{1} = \frac{3b}{2}$

3. $\frac{15x^2y^3}{5xy - 10y} \cdot \frac{x^2 - 4}{3x^2 + 6x}$

solution:

* $\frac{15x^2y^3}{5xy - 10y} \cdot \frac{x^2 - 4}{3x^2 + 6x} = \frac{15x^2y^3}{5y(x-2)} \cdot \frac{(x-2)(x+2)}{3x(x+2)}$

$= xy^2$

Related Problem (2)

4. $\frac{x+y}{x^2 - y^2} \div \frac{x^2 - xy}{x^2 - 2xy + y^2}$, $x \neq \pm y$

solution:

* $\frac{x+y}{x^2 - y^2} \div \frac{x^2 - xy}{x^2 - 2xy + y^2} = \frac{x+y}{x^2 - y^2} \cdot \frac{x^2 - 2xy + y^2}{x^2 - xy}$

$= \frac{x+y}{(x+y)(x-y)} \cdot \frac{(x-y)^2}{x(x-y)}$

$= \frac{1}{x}$

Example 2

5. $\frac{2x^3 - 2x^2y + 2xy^2}{x^3y - xy^3} \div \frac{x^3 + y^3}{x^2 + 2xy + y^2}$, $y \neq \pm x$, $x \neq 0$, $y \neq 0$

solution:

* $\frac{2x^3 - 2x^2y + 2xy^2}{x^3y - xy^3} \cdot \frac{x^2 + 2xy + y^2}{x^3 + y^3} = \frac{2x(x^2 - xy + y^2)}{xy(x^2 - y^2)} \cdot \frac{(x+y)^2}{(x+y)(x^2 - xy + y^2)}$

$= \frac{2x(x^2 - xy + y^2)}{xy(x-y)(x+y)} \cdot \frac{(x+y)^2}{(x+y)(x^2 - xy + y^2)}$

$= \frac{2}{y(x-y)}$

Related Problem 3

1. $\frac{2}{15}+\frac{13}{10}-\frac{7}{6}$

solution:

نبحث عن المضاعف المشترك الأصغر . و نضرب كل كسر بعدد بسطا و مقاما لتوحيد المقامات 30 و جمعها

L.C.M of 15 , 10 , 6 is 30

$$* \quad \frac{2}{15}+\frac{13}{10}-\frac{7}{6}=\frac{2\cdot 2}{15\cdot 2}+\frac{13\cdot 3}{10\cdot 3}-\frac{7\cdot 5}{6\cdot 5}$$

$$=\frac{4}{30}+\frac{39}{30}+\frac{35}{30}$$

$$=\frac{4+39-35}{30}$$

$$=\frac{8}{30}=\frac{4}{15}$$

فكرة مثال

2. $\frac{4}{3x}+\frac{2x}{5y^2}+3 \quad , \quad x\neq 0 \quad , \quad y\neq 0$

solution:

L.C.M of $3x$ and $5y^2$ is $15xy^2$

$$* \quad \frac{4}{3x}+\frac{2x}{5y^2}+3=\frac{4}{3x}\cdot\frac{5y^2}{5y^2}+\frac{2x}{5y^2}\cdot\frac{3x}{3x}+\frac{3}{1}\cdot\frac{15xy^2}{15xy^2}$$

$$=\frac{20y^2+6x^2+15xy^2}{15xy^2}$$

Exercise 5: $\frac{x+2}{x^2-1}-\frac{x-2}{(x-1)^2} \quad ; x\neq \pm 1$

solution:

نوحد المقامات لكل كسر ثم نطرح

$$* \quad \frac{x+2}{x^2-1}-\frac{x-2}{(x-1)^2}=\frac{x+2}{(x-1)(x+1)}-\frac{x-2}{(x-1)^2}$$

$$=\frac{(x+2)}{(x-1)(x+1)}\cdot\frac{(x-1)}{(x-1)}-\frac{(x-2)(x+1)}{(x-1)^2(x+1)}$$

$$=\frac{(x+2)(x-1)-(x-2)(x+1)}{(x-1)^2(x+1)}$$

$$=\frac{(x^2+x-2)-(x^2-x-2)}{(x-1)^2(x+1)}$$

$$=\frac{x^2+x-2-x^2+x+2}{(x-1)^2(x+1)}=\frac{2x}{(x-1)^2(x+1)}$$

Exercise 6. $\frac{4x}{x^2-y^2}+\frac{3}{x+y}-\frac{2}{x-y}$; $x \neq \pm y$

solution:

* $\frac{4x}{x^2-y^2}+\frac{3}{x+y}-\frac{2}{x-y}=\frac{4x}{(x-y)(x+y)}+\frac{3(x-y)}{(x+y)(x-y)}-\frac{2(x+y)}{(x-y)(x+y)}$

$$=\frac{4x+3(x-y)-2(x+y)}{(x-y)(x+y)}$$

$$=\frac{4x+3x-3y-2x-2y}{(x-y)(x+y)}$$

$$=\frac{5x-5y}{(x-y)(x+y)}$$

$$=\frac{5(x-y)}{(x-y)(x+y)}$$

$$=\frac{5}{x+y}$$

Exercise 10. $\frac{1}{y^2+y}-\frac{1}{y^2-1}-\frac{1}{y}$; $y \neq -1,0,1$

solution:

* $\frac{1}{y^2+y}-\frac{1}{y^2-1}-\frac{1}{y}=\frac{1}{y(y+1)}-\frac{1}{(y-1)(y+1)}-\frac{1}{y}$

$$=\frac{1}{y(y+1)}\cdot\frac{(y-1)}{(y-1)}-\frac{1}{(y-1)(y+1)}\cdot\frac{y}{y}-\frac{1}{y}\cdot\frac{(y-1)(y+1)}{(y-1)(y+1)}$$

$$=\frac{y-1-y-(y^2-1)}{y(y+1)(y-1)}$$

$$=\frac{-y^2}{y(y+1)(y-1)}$$

$$=\frac{-y}{(y+1)(y-1)}$$

Ecercise 7. $\dfrac{\dfrac{x^2}{y^2}-1}{\dfrac{x}{y}+1}$; $y \neq 0$, $\dfrac{x}{y} \neq -1$

solution:

$$* \quad \frac{\dfrac{x^2}{y^2}-1}{\dfrac{x}{y}+1} = \frac{\dfrac{x^2-y^2}{y^2}}{\dfrac{x+y}{y}}$$

$$= \frac{x^2-y^2}{y^2} \div \frac{x+y}{y}$$

$$= \frac{(x-y)(x+y)}{y^2} \cdot \frac{y}{x+y}$$

$$= \frac{x-y}{y}$$

Exercise 15. $1-\dfrac{1}{1-\dfrac{1}{1-\dfrac{1}{x}}}$; $x \neq 0,1$

solution:

$$* \quad 1-\frac{1}{1-\dfrac{1}{1-\dfrac{1}{x}}} = 1-\frac{1}{1-\dfrac{1}{\dfrac{x-1}{x}}}$$

توحيد المقامات من أسفل

$$= 1-\frac{1}{1-\dfrac{x}{x-1}}$$

$$= 1-\frac{1}{\dfrac{x-1-x}{x-1}}$$

$$= 1-\frac{1}{\dfrac{-1}{x-1}}$$

$$= 1+(x-1) = x$$

Exercise 9. $\dfrac{2x(1-3x)^3+9x^2(1-3x)^2}{(1-3x)^6}$; $x \neq \dfrac{1}{3}$

نأخذ $x(1-3x)^2$ عامل مشترك من البسط

solution:

$$\frac{2x(1-3x)^3+9x^2(1-3x)^2}{(1-3x)^6}=\frac{x(1-3x)^2\left(2(1-3x)+9x\right)}{(1-3x)^6}$$

$$=\frac{x\left(2-6x+9x\right)}{(1-3x)^4}$$

$$=\frac{x\left(3x+2\right)}{(1-3x)^4}$$

Imaginary unit :

* $\sqrt{-1} = i$
* $\sqrt{-4} = \sqrt{-1 \cdot 4} = \sqrt{4} \cdot \sqrt{-1} = 2i$
* $\sqrt{-25} = \sqrt{25 \cdot -1} = 5i$

A complex number in standard form is $a + bi$

Where a is the real part , b is the imaginary part

• نعطي مثال لتوضيح الجزء الحقيقي و التخيلي و تمثيله

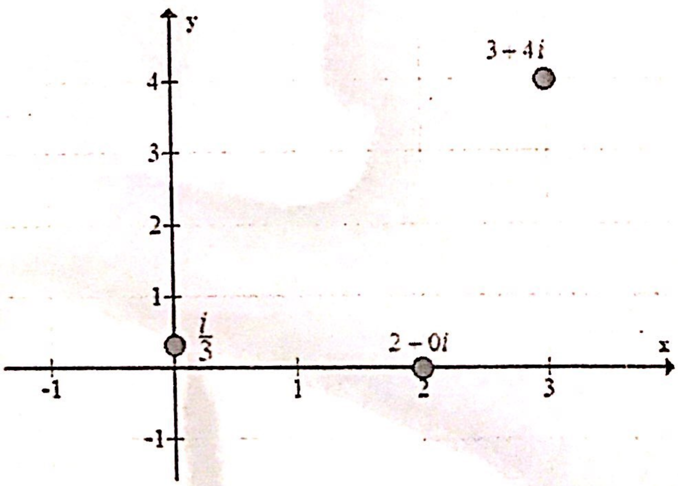
Example : Identify the real and imaginary part of each the following complex numbers.

حدد الجزء الحقيقي و الجزء التخيلي لكل عدد مركب

1. $3+4i$ 2. $\frac{i}{3}$ 3. 2 4. $7+\sqrt{-36}$

solution:

Complex Number z	Real part : $\mathrm{Re}(z)$	Imaginary Part : $\mathrm{Im}(z)$	Conjugate $\bar{z}$
$3+4i$	3	4	$3-4i$
$\frac{i}{3}$	0	$\frac{1}{3}$	$-\frac{i}{3}$
2	2	0	2
$7+\sqrt{-36} = 7+6i$	7	6	$7-6i$



Example: Find the values of x and y are real numbers, that satisfy the given equation

أوجد قيم x و y الأعداد الحقيقية التي تحقق المعادلات المعطاة

Ecercise 1. $4+(2y)i=x-2i$

solution:

real part , imaginary part

$x=4$ $\quad$ $2y=-2$

$y=-1$

Exercise 3. $2yi=x+12i$

solution:

$0+2yi=x+12i$

Real part : $x=0$

Imaginary : $2y=12$, $y=6$

Example: Perform each the following operations, and write your answer in standard form

نفذ العمليات التالية و اكتب اجابتك في الصيغة القياسية $a+bi$

Exercise 13. $(3-2i)+(5+2i)$

عند جمع الأعداد المركبة نجمع الجزء الحقيقي مع الجزء الحقيقي . و الجزء التخيلي مع الجزء التخيلي

solution:

* $(3-2i)+(5+2i)=(3+5)+(-2i+2i)$

$=8+0i$

Exercise 15. $\left(8+\frac{3}{4}i\right)+\left(-7+\frac{2}{3}i\right)$

solution:

* $\left(8+\frac{3}{4}i\right)+\left(-7+\frac{2}{3}i\right)=(8-7)+\left(\frac{3}{4}i+\frac{2}{3}i\right)$

$=1+\frac{17}{12}i$

16. $\left(3+\frac{3}{5}i\right)-\left(-11+\frac{7}{15}i\right)$

solution:

* $\left(3+\frac{3}{5}i\right)-\left(-11+\frac{7}{15}i\right)=3+\frac{3}{5}i+11-\frac{7}{15}i$

$=14+\frac{2}{15}i$

Exercise 17. $(3i)\cdot(-5i)$

solution:

* $(3i)\cdot(-5i)=-15i^2$

$=-15(-1)=15$

$\boxed{i^2=-1}$

Exercise 19. $3(2-3i)$

solution:

* $3(2-3i)=6-9i$

20. $-3i\cdot(7+5i)$

solution:

* $-3i\cdot(7+5i)=-21i-15i^2$

$=-21i+15=15-21i$

Exercise 21. $(-3+2i)\cdot(2+3i)$

solution:

* $(-3+2i)\cdot(2+3i)=-6-9i+4i-6$

$=-12-5i$

Exercise 23. $\left(1+\frac{1}{2}i\right)\cdot\left(\frac{1}{2}+\frac{2}{3}i\right)$

solution:

* $\left(1+\frac{1}{2}i\right)\cdot\left(\frac{1}{2}+\frac{2}{3}i\right)=\frac{1}{2}+\frac{2}{3}i+\frac{1}{4}i-\frac{1}{3}$

$=\frac{1}{6}+\frac{11}{12}i$

Exercise 25. $(2-\sqrt{-4})\cdot\left(3-\sqrt{-16}\right)$

solution:

* $(2-\sqrt{-4})\cdot\left(3-\sqrt{-16}\right)=(2-2i)(3-4i)$

$=6-8i-6i-8$

$=-2-14i$

Exercise 29. $i(2-i^3)$

solution:

* $i(2-i^3)=2i-i^4$

$=2i-1=-1+2i$

$* \; i^4=i^2\cdot i^2=(-1)(-1)=1$

24. $(3-i\sqrt{2})(3+i\sqrt{2})$

solution:

* $(3-i\sqrt{2})(3+i\sqrt{2})=(3)^2+(\sqrt{2})^2$

$=11$

ضرب العدد المركب بمرافقه
$(a+bi)(a-bi)=a^2+b^2$

Example : Write in standard form

تم ترتيب الأسئلة المتشابهة معا

Exercise 5. $\dfrac{2+\sqrt{-25}}{4}$

solution:

* $\dfrac{2+\sqrt{-25}}{4}=\dfrac{2+5i}{4}$

$=\dfrac{2}{4}+\dfrac{5}{4}i=\dfrac{1}{2}+\dfrac{5}{4}i$

$* \; \sqrt{-25}=\sqrt{(25)(-1)}=5\sqrt{-1}=5i$

Exercise 9. $\dfrac{8+\sqrt{-27}}{6}$

solution:

* $\dfrac{8+\sqrt{-27}}{6}=\dfrac{8+\sqrt{-9\cdot 3}}{6}$

$=\dfrac{8+3\sqrt{3}i}{6}$

$=\dfrac{8}{6}+\dfrac{3\sqrt{3}}{6}i=\dfrac{4}{3}+\dfrac{\sqrt{3}}{2}i$

Exercise 10. $\dfrac{\sqrt{-36}\sqrt{-49}}{\sqrt{-16}}$

solution:

* $\dfrac{\sqrt{-36}\sqrt{-49}}{\sqrt{-16}}=\dfrac{6i\cdot 7i}{4i}$

$=\dfrac{42}{4}i=\dfrac{21}{2}i$

Exercise 11. $\dfrac{\sqrt{-25}}{\sqrt{-16}\sqrt{-81}}$

solution:

* $\dfrac{\sqrt{-25}}{\sqrt{-16}\sqrt{-81}}=\dfrac{5i}{4i\cdot 9i}$

$=\dfrac{5i}{36i^2}$

$=\dfrac{5i}{-36}=-\dfrac{5}{36i}$

Exercise 31. $\dfrac{3}{1+3i}$

solution:

* $\dfrac{3}{1+3i}=\dfrac{3}{1+3i}\cdot\dfrac{1-3i}{1-3i}$

$=\dfrac{3-9i}{(1)^2+(3)^2}$

$=\dfrac{3}{10}-\dfrac{9}{10}i$

ضرب العدد المركب بمرافقه

$(a+bi)(a-bi)=a^2+b^2$

Exercise 32. $\dfrac{2+3i}{3i}$

solution:

* $\dfrac{2+3i}{3i}=\dfrac{2+3i}{3i}\cdot\dfrac{-3i}{-3i}$

$=\dfrac{-6i-9i^2}{-9i^2}$

$=\dfrac{9-6i}{9}$

$=1-\dfrac{2}{3}i$

Exercise 34. $\dfrac{3-2i}{-6+4i}$

solution:

* $\dfrac{3-2i}{-6+4i}=\dfrac{3-2i}{-2(3-2i)}$

$=-\dfrac{1}{2}+0i$

اذا حليت المثال بالضرب بالمرافق سيعطي نفس الناتج

36. $\dfrac{-4+6i}{2+7i}$

solution:

* $\dfrac{-4+6i}{2+7i} = \dfrac{-4+6i}{2+7i} \cdot \dfrac{2-7i}{2-7i}$

$$= \frac{-8+28i+12i+42}{(2)^2+(7)^2}$$

$$= \frac{34+40i}{53}$$

$$= \frac{34}{53} + \frac{40}{53}i$$

Notes:

* $i = \sqrt{-1}$
* $i^2 = -1$
* $i^3 = i^2 \cdot i = -i$
* $i^4 = i^2 \cdot i^2 = (-1)\cdot(-1) = 1$

Example : Evaluate

وضعت جميع الأفكار في الأمثلة و التمارين

Exercise 37. i^{79}

solution:

* $i^{79} = i^{76} \cdot i^3$

$$= \left(i^4\right)^{19} \cdot (-i) = -i$$

Exercise 38. i^{-11}

solution:

* $i^{-11} = \dfrac{1}{i^{11}}$

$$= \frac{1}{(i^4)^2 \cdot i^3}$$

$$= \frac{1}{-i} \cdot \frac{i}{i}$$

$$= \frac{i}{-i^2} = \frac{i}{-(-1)} = i$$

Exercise 39. i^{14}

solution:

* $i^{14} = i^{12} \cdot i^2$

$= (i^4)^3 \cdot (-1)$

$= (1)^3(-1) = -1$

Exercise 40. i^{23}

solution:

* $i^{23} = i^{20} \cdot i^3$

$= (i^4)^5 \cdot (-i)$

$= (1)^5(-1) = -i$

Exercise 41. i^{-6}

solution:

* $i^{-6} = \frac{1}{i^6}$

$= \frac{1}{i^4 \cdot i^2}$

$= \frac{1}{(1)(-1)} = -1$

Ex. i^{32}

solution:

* $i^{32} = \left(i^4\right)^8$

$= (1)^8 = 1$

E

Exercise 42 : Show that

a. Show that $\overline{z+3i} = z - 3i$

solution:

* $\overline{z+3i} = \overline{\overline{z}} - 3i$

$= z - 3i$

b. Show that $\overline{iz} = -i\,\overline{z}$

solution:

* $\overline{iz} = (\overline{i})(\overline{z})$

$= (-i)\overline{z} = -\overline{z}$

c. Show that $\overline{(2-i)^2} = 3+4i$

solution:

$(2-i)^2 = (2)^2 - 2(2)(i) + (i)^2 = 4 - 4i - 1 = 3 - 4i$

* $\overline{(2-i)^2} = \overline{3-4i} = 3+4i$

Exercise 43: Let $z_1 = 4-3i$, $z_2 = 5-3i$, $z_3 = -2i$, find:

a. $\text{Re}(z_1)$, $\text{Im}(z_3)$, $\text{Re}(z_1 z_2)$

solution:

$\text{Re}(z_1) = 4$

$\text{Im}(z_3) = -2$

$z_1 z_2 = (4-3i)(5-3i) = 20 - 12i - 15i - 9 = 11 - 27i$

$\text{Re}(z_1 z_2) = 11$

b. $z_1 - z_2$

solution:

* $z_1 - z_2 = (4-3i) - (5-3i)$

$= 4 - 3i - 5 + 3i$

$= -1 + 0i$

c. $\frac{z_1}{z_2}$

soution:

* $\frac{z_1}{z_2} = \frac{4-3i}{5-3i} \cdot \frac{5+3i}{5+3i}$

$= \frac{20+12i-15i+9}{25+9}$

$= \frac{29-3i}{34}$

$= \frac{29}{34} - \frac{3}{34}i$

d. z_3^{-1}

solution:

* $z_3^{-1} = \frac{1}{z_3}$

$= \frac{1}{-2i} \cdot \frac{2i}{2i}$

$= \frac{2i}{-4i^2} = \frac{2i}{-4(-1)}$

$= \frac{1}{2}i = 0 + \frac{1}{2}i$

e. $3i^{34} - z_3^3$

solution:

* $3i^{34} - z_3^3 = 3(i^{32})(i^2) - (-2i)^3$

$= 3(1)(-1) - (-8i^3)$

$= -3 - (-8 \cdot -i)$

$= -3 - 8i$

f. $z_1\overline{z_1}$

solution:

* $z_1\overline{z_1} = (4-3i)(4+3i)$

$= (4)^2 + (3)^2$

$= 25 + 0i$

SECTION (2 – 1) : LINEAR EQUATIONS AND APPLICATIONS

المعادلات الخطية و تطبيقاتها

A linear equation in one variable has the standard form $ax + b = 0$
where $a, b \in \mathbb{R}$, $a \neq 0$

For example : $3x + 2 = 0$, $\frac{1}{3}y = 4$, $3(x - 2) = 0$

Example: Solve each of the following equations and check your answer

حل كل من المعادلات التالية و تأكد من حلك

Exercise 1. $5x = 3x - (1 - 3x)$.

solution:

$$5x = 3x - 1 + 3x$$
$$5x = 6x - 1$$
$$5x - 6x = -1$$
$$-x = -1 \qquad \text{(divide by } -1\text{)}$$
$$x = 1$$

عند حل المعادلات الخطية
نضع المجاهيل بطرف و الأعداد بطرف

The solution is $x = 1$

* Check:

L.H.S: $5(1) = 5$

R.H.S: $3(1) - (1 - 3(1)) = 3 - (-2) = 5$

Exercise 2. $4(2y - 17) + 5(3y - 8) = 0$

solution:

$$8y - 68 + 15y - 40 = 0$$

نوزع الأعداد على الأقواس و نجمع المتشابه

$$23y - 108 = 0$$
$$23y = 108 \qquad \text{divide by } 23$$
$$y = \frac{108}{23}$$

The solution is $y = \frac{108}{23}$

* Check

L.H.S $= 4(2(\frac{108}{23}) - 17) + 5(3(\frac{108}{23}) - 8)$

$= 4\left(-\frac{175}{23}\right) + 5\left(\frac{140}{23}\right)$

$= 0 = \text{R.H.S}$

Exercise 4. $\frac{1}{2}x+5=\frac{1}{3}x+7$

solution: multiply all by 6

$$(6)\frac{1}{2}x+(6)5=(6)\frac{1}{3}x+(6)7$$
$$3x+30=2x+42$$
$$3x-2x=42-30$$
$$x=12$$

* Check

L.H.S: $\frac{1}{2}(12)+5=6+5=11$

R.H.S: $\frac{1}{3}(12)+7=4+7=11$

Another solution $\frac{1}{2}x+5=\frac{1}{3}x+7$

solution:

$$\frac{x}{2}+5=\frac{x}{3}+7$$
$$\frac{x+10}{2}=\frac{x+21}{3}$$
$$3(x+10)=2(x+21)$$
$$3x+30=2x+42$$
$$3x-2x=42-30$$
$$x=12$$

The solution is $x=12$ and check

Exercise 8. $\frac{x}{2}+\frac{2x-1}{3}=\frac{3x+4}{4}$

solution

$$\frac{3x+2(2x-1)}{6}=\frac{3x+4}{4}$$

توحيد المقامات للطرف الأيسر

$$\frac{3x+4x-2}{6}=\frac{3x+4}{4}$$
$$\frac{7x-2}{6}=\frac{3x+4}{4}$$
$$4(7x-2)=6(3x+4)$$

حاصل ضرب الطرفين = ضرب الوسطين

$$28x-8=18x+24$$
$$28x-18x=24+8$$
$$10x=32$$
$$x=\frac{32}{10}=\frac{16}{5}$$

* Check

L.H.S: $\frac{\frac{16}{5}}{2}+\frac{2(\frac{16}{5})-1}{3}=\frac{16}{10}+\frac{9}{5}=\frac{17}{5}$

R.H.S: $\frac{3(\frac{16}{5})+4}{4}=\frac{17}{5}$

Example 3 : The price of a company stock has been increased by 10% and is being sold for 99 SR . Find the original price of the stock

solution:

سعر أسهم شركة زادت بنسبة 10% ليصبح سعرها 99 ريال . اوجد السعر الأصلي للأسهم

Let the original price is P and increased rate 10% ($0.10P$)

* The price after increasing = original price + (increasing rate)(original price)

Then the new price $99 = P + 0.10P$

$$99 = 1.01P$$

$$P = \frac{99}{1.01} = 90$$

the original price is 90 SR

Example 4:The book store in the preparatory deanship in KSU announced 35% discount for Math 140 book which worth 78 SR after discount . Find the original price

solution:

مكتبة عمادة السنة التحضيرية بجامعة الملك سعود اعلنت عن خصم 35% لكتاب ريض 140 ليعادل قيمة 78 ريال بعد الخصم . اوجد السعر الأصلي

Let the orignal price P , discount 35% ($0.35P$)

* Price after discount = original price – (discount rate)(the original price)

$$78 = P - 0.35P$$

$$78 = 0.65P$$

$$\frac{78}{0.65} = P \quad \Rightarrow \quad P = 120$$

The original price is 120 SR

Exercise 19 : The sale price of camera after a 20% discount is SR 72 . What was the price before the discount?

solution:

سعر البيع لكاميرا بعد خصم 20% هو 72 ريال . ما السعر قبل الخصم

Let the original price is P , discount 20% ($0.20P$)

* The price after discount = the original price – (discount rate) · (the original price)

$$72 = P - 0.2P$$

$$72 = 0.8P \qquad \text{divide by } (0.8)$$

$$P = 90$$

The original price is SR 90

Example 5: Given four consecutive even integers . the sum of the first three exceeds the fourth by 8 . Find these numbers

solution:

اربع اعداد صحيحة زوجية متتالية . مجموع أول ثلاثة أعداد تزيد عن الرابع بـ 8 . أوجد هذه الأعداد

Let the numbers are x , $x+2$, $x+4$, $x+6$

* The linear equation $(x)+(x+2)+(x+4)=(x+6)+8$

$$3x+6=x+14$$

$$3x-x=14-6$$

$$2x=8 \quad , \quad x=4$$

The numbers are 4 , 6 , 8 and 10

Exercise 13 : Find two consecutive odd integers such that three times the smaller one exceeds two times the larger one by 7

solution

أوجد عددين صحيحين فرديين متتالين . بحيث ثلاثة أضعاف الأصغر يزيد عن ضعف الأكبر بـ 7

Let the two consecutive odd integers are x and $x+2$

* We have a linear equation

$$3(x)=2(x+2)+7$$

$$3x=2x+4+7$$

$$3x-2x=11$$

$$x=11$$

The two consecutive odd integers are 11 and 13

Example6 : A rectangular land has a perimeter 84 meters . If the length is 3 meters less than twice the width , find the dimentions of the rectangular (length and width)

solution:

أرض مستطيلة محيطها 84 . اذا كان طولها يقل عن ضعف العرض بـ 3 .أوجد أبعاد المستطيل (الطول و العرض)

Let the width is x and the length is $y=2x-3$

* Perimeter of rectangular $=2(\text{length})+2(\text{width})$

$$2(2x-3)+2(x)=84$$

$$4x-6+2x=84$$

$$6x=84+6=90$$

$$x=\frac{90}{6}=15$$

Then the width of rectangular is 15 and the length is $2(15)-3=27$

Exercise 14 : If the width of rectangle is 5 cm more than one-half its length and the perimeter is 46 cm . What are the dimensions of the rectangle

اذا كان عرض مستطيل اكبر من نصف الطول بـ 5 و محيطه 46 . أوجد ابعاد المستطيل

solution:

Length of the rectangle is y and the width $x = \frac{1}{2}y + 5$

* Perimetre of rectangle P = 2(length) + 2(width)

$$2y + 2(\frac{1}{2}y + 5) = 46$$

$$2y + y + 10 = 46$$

$$3y = 36$$

$$y = 12$$

$$x = \frac{1}{2}y + 5 = \frac{1}{2}(12) + 5 = 11$$

The length is 12 , The width is 11

SECTION (2 – 2) : LINEAR INEQUALITIES المتباينات الخطية

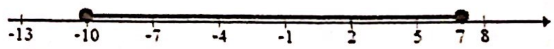
Example: Rewrite the following intervals in inequality notation and graph it on real number line.

أعد كتابة الفترة في صيغة المتباينة و مثلها على خط الأعداد

Exercise 1. $[-10,7]$

solution:

inequality notation: $-10 \le x \le 7$



Exercise 2. $(-4,12)$

solution:

inequality notation: $-4 < x < 12$

Exercise 3. $(-\infty,8]$

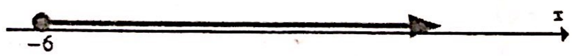
solution:

inequality notation: $x \le 8$

Exercise 4. $[-6,\infty)$

solution:

inequalties notation: $x \ge -6$

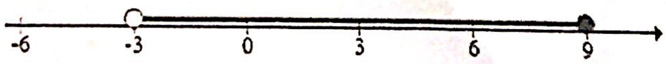


Example: Rewrite in interval notation and graph it on real number line

Exercise 5. $-3 < x \le 9$

solution:

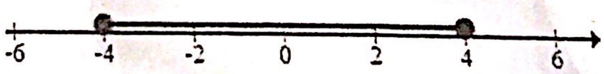
interval notation: $(-3,9]$



Exercise 6. $-4 \le x \le 4$

solution:

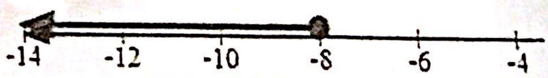
interval notation: $[-4,4]$



Exercise 7. $x \le -8$

solution:

interval notation: $(-\infty,-8]$



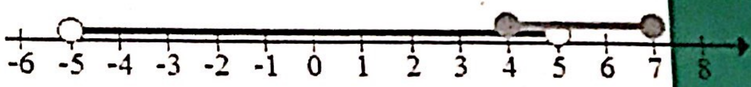
Graph the indicated sets and write it as a single interval , if possible.

مثل المجموعات المعطاة و اكتبها كـ فترة واحدة ان امكن

Exercise 9. $(-5,5)\cup[4,7]$

solution:

اتحاد فترتين أي دمجهم و نأخذ من البداية للنهاية

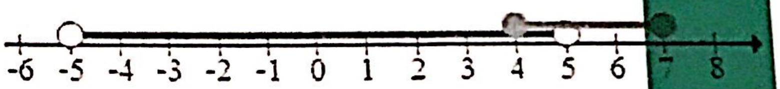


* $(-5,5)\cup[4.7]=(-5,7]$

Exercise 10. $(-5,5)\cap[4,7]$

solution:

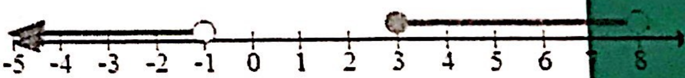
تقاطع فترتين أي الفترة المشتركة بين الفترتين



* $(-5,5)\cap[4.7]=[4,5)$

Exercise 12. $(-\infty,-1)\cup[3,8)$

solution:



* $(-\infty,-1)\cup[3.8)=(-\infty,-1)\cup[3.8)$

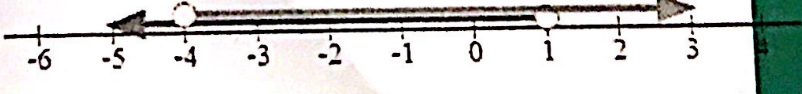
Exercise 13. $[2,3]\cup(1,5)$

solution:

* $[2,3]\cup(1.5)=(1,5)$

Exercise 14. $(-\infty,1)\cap(-4,\infty)$

solution:



* $(-\infty,1)\cap(-4.\infty)=(-4,1)$

Example: Solve and graph the following inequalities

حل و مثل المتباينات التالية

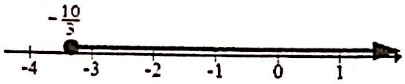
Exercise 16. $4x+8 \geq x-2$

solution:

$$4x-x \geq -2-8$$

$$3x \geq -10$$

$$x \geq -\frac{10}{3}$$



The solution set is $\left[-\frac{10}{3},\infty\right)$

Exercise 18. $-4<3x+6 \leq 4x-5$

solution:

$-4<3x+6$	and	$3x+6 \leq 4x-5$
$-4-6<3x$		$3x-4x \leq -5-6$
$-10<3x$		$-x \leq -11$
$-\frac{10}{3}<x$		$x \geq 11$

??????????????

The solution set is $\left(-\frac{10}{3},\infty\right)\cap[11,\infty)=[11,\infty)$

Exercise 20. $\frac{y-3}{4}-2>\frac{y}{3}+2$

solution: multiply all by $(3)(4)$

$$(3)(4)\frac{y-3}{4}-(3)(4)2>(3)(4)\frac{y}{3}+(3)(4)2$$

$$3(y-3)-24>4y+24$$

$$3y-9-24>4y+24$$

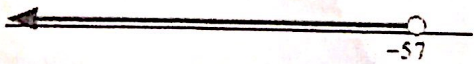
$$3y-4y>24+33$$

$$-y>57$$

$$y<-57$$

عند الضرب أو القسمة على عدد سالب نعير علامة المتباينة

The solution set is the interval $(-\infty,-57)$



Exercise 24. $15 \le 7-\frac{2}{5}x \le 21$

solution:

$$15-7 \le -\frac{2}{5}x \le 21-7$$

$$8 \le -\frac{2}{5}x \le 14 \qquad \text{multiply all by } -\frac{5}{2}$$

عند ضرب او قسمة المتباينة بعدد سالب تتغير علامة المتباينة

$$-20 \ge x \ge -35$$

The solution set is the interval $[-35,-20]$

Exercise 25. $\frac{2x}{5}-\frac{1}{2}(x-3) \le \frac{2x}{3}-\frac{3}{10}(x+2)$

solution: multiply all by (30)

$$(30)\frac{2x}{5}-(30)\frac{1}{2}(x-3) \le (30)\frac{2x}{3}-(30)\frac{3}{10}(x+2)$$

$$12x-15(x-3) \le 20x-9(x+2)$$

$$12x-15x+45 \le 20x-9x-18$$

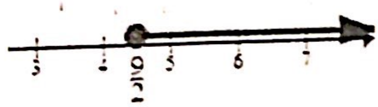
$$-3x+45 \le 11x-18$$

$$-3x-11x \le -18-45$$

$$-14x \le -63$$

$$x \ge \frac{9}{2}$$

The solution set is the interval $\left[\frac{9}{2},\infty\right)$



Example 4

Ex: In a chemistry experiment a solution of hydrochloric acid is to be kept between 30° C and 35°C, that is $30 \le C \le 35$. What is the range in temprature in degrees Fahrenheit if Celsius\Fahreheit conversion $C=\frac{5}{9}(F-32)$?

Solution:

$$30 \le C \le 35$$

$$30 \le \frac{5}{9}(F-32) \le 35 \qquad \text{multiply all by } \frac{9}{5}$$

$$\frac{9}{5}\cdot 30 \le \frac{9}{5}\cdot\frac{5}{9}(F-32) \le \frac{9}{5}\cdot 35$$

$$54 \le (F-32) \le 63$$

$$54+32 \le F \le 63+32$$

$$86 \le F \le 95$$

The range of tempreture is from $86\ F$ to $95\ F$

Example 5

Ex : A film developer is to be kept between $68°F$ and $77°F$, that is $68 \le F \le 77$

What is the range in tempreture in degree Celsius/Fahrenheit conversion formula is $F = \frac{9}{5}C + 32$?

solution:

$$68 \le F \le 77$$

$$68 \le \frac{9}{5}C + 32 \le 77$$

$$68 - 32 \le \frac{9}{5}C \le 77 - 32$$

$$36 \le \frac{9}{5}C \le 45 \qquad \text{multiply all by } \frac{5}{9}$$

$$\frac{5}{9} \cdot 36 \le \frac{5}{9} \cdot \frac{9}{5}C \le \frac{5}{9} \cdot 45$$

$$20 \le C \le 25$$

The range of the tempreture is from $20°C$ to $25°C$

SE

Ex

*

SECTION (2 – 3) : EQUATIONS AND INEQUALITIES INVOLVING ABSOLUTE VALUE القيمة المطلقة في المعادلات و المتباينات

Example:- Evaluate the following

1. $|7| = 7$

2. $|-6| = 6$

3. $|-\pi| = \pi$

4. $|-3+\sqrt{5}| = -(-3+\sqrt{5}) = 3-\sqrt{5}$

Definition : Absolute value of a real number

$$* \quad |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$* \quad |-x| = \begin{cases} -x & \text{if } -x \geq 0 \\ -(-x) & \text{if } -x < 0 \end{cases} = \begin{cases} -x & \text{if } x \leq 0 \\ x & \text{if } x > 0 \end{cases}$$

لتسهيل حل معادلات ومتباينات القيمة المطلقة (absolute value) نقسمها الى ثلاث قواعد

(1) Equation → $|x| = a$

(2) inequality (less than) → $|x| < a$

(3) inequality (more than) → $|x| > a$

1- Solving absolute value equations (الحالة الاولى : القيمة المطلقة في المعادلة)

$$|x| = a$$

$x = a$ or $x = -a$

Example : Solve each equation

مثال لتوضيح النظرية

1. $|x| = 5$

solution:

$x = 5$ or $x = -5$

The solution set is $\{-5, 5\}$

Example

2. $|2x + 5| = 0$

solution:

$2x + = 0$, $2x = -5$, $x = \frac{-5}{2}$

The solution set is $\left\{\frac{-5}{2}\right\}$

3. $|x| = -3$

solution:

$|x| = -3$ has no solution

Exercise 1. $|2x + 6| = 10$

solution:

The solutions are $2x + 6 = \pm 10$

$2x + 6 = 10$ or $2x + 6 = -10$

$2x = 4$ or $2x = -16$

$x = 2$ or $x = -8$

The solution set is $\{-8, 2\}$

Exercise 2. $-3|x+5|+6=-15$

solution:

$-3|x+5|=-15-6$

$-3|x+5|=-21$ divide by (-3)

$|x+5|=7$

The solutions are $x+5=\pm 7$

$x+5=7$ or $x+5=-7$

$x=2$ or $x=-12$

The solution set is $\{-12,2\}$

Exercise 7. $\left|\frac{1}{3}y+\frac{5}{6}\right|=1$

solution:

The solutions are $\frac{1}{3}y+\frac{5}{6}=\pm 1$

$\frac{1}{3}y+\frac{5}{6}=1$ or $\frac{1}{3}y+\frac{5}{6}=-1$

$\frac{1}{3}y=1-\frac{5}{6}$ or $\frac{1}{3}y=-1-\frac{5}{6}$

$\frac{1}{3}y=\frac{1}{6}$ or $\frac{1}{3}y=-\frac{11}{6}$ multiply all by (3)

$y=\frac{1}{2}$ or $y=-\frac{11}{2}$

The solution set is $\left\{-\frac{11}{2}, \frac{1}{2}\right\}$

Exercise 4. $|7-3x|=2x+5$

solution:

$|3x-7|=2x+5$

$|3x-7|\geq 0$ then $2x+5\geq 0$, $2x\geq -5$, $x\geq \frac{-5}{2}$ نبحث عن الفترة التي يقع بها الحل

$|3x-7|=2x+5$, $x\geq -\frac{5}{2}$

$3x-7=\pm(2x+5)$

$3x-7=2x+5$ or $3x-7=-(2x+5)=-2x-5$

$3x-2x=5+7$ or $3x+2x=-5+7$

$x=12$ or $5x=2$, $x=\frac{2}{5}$

Remark: 12 and $\frac{2}{5} > -\frac{5}{2}$

The solution set is $\left\{\frac{2}{5}, 12\right\}$

Exercise 5. $|2x+3| = x-1$

solution:

$|2x+3| \ge 0$ then $x-1 \ge 0$, $x \ge 1$

* $|2x+3| = x-1$, $x \ge 1$

$2x+3 = \pm(x-1)$

$2x+3 = x-1$ or $2x+3 = -(x-1) = -x+1$

$2x-x = -1-3$ or $2x+x = 1-3$

$x = -4$ or $3x = -2$, $x = -\frac{2}{3}$

but $x = -4$ and $x = -\frac{2}{3}$ <1

The equation has no solution

2- Solving absolute value with <u>less than</u> inequality(الحالة الثاني:القيمة المطلقة في متباينة <u>أقل من</u>)

تكافئ

$|f(x)| < a$ is equivalent to $-a < f(x) < a$

Example : Solve the following inequalities and graph the solution set :

مثال لتوضيح القاعدة

1. $|x| \le 3$

solution:

$-3 \le x \le 3$

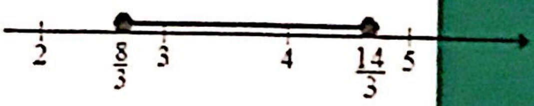
The solution set is $[-3,3]$

Exercise 11. $|3x-11|+6\le 9$

solution

$$|3x-11|\le 9-6$$
$$|3x-11|\le 3$$
$$-3\le 3x-11\le 3$$
$$-3+11\le 3x\le 3+11$$
$$8\le 3x\le 14$$
$$\frac{8}{3}\le x\le \frac{14}{3}$$

The solution set is the interval $\left[\frac{8}{3},\frac{14}{3}\right]$

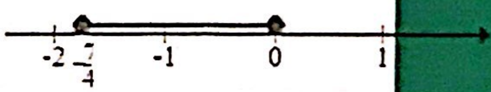


Exercise 12. $\left|\frac{4y+5}{3}-\frac{1}{2}\right|\le\frac{7}{6}$

solution: multiply all by (6)

$$-\frac{7}{6}\cdot 6\le 6\cdot\frac{4y+5}{3}-6\cdot\frac{1}{2}\le 6\cdot\frac{7}{6}$$
$$-7\le 2(4y+5)-3\le 7$$
$$-7+3\le 2(4y+5)\le 7+3$$
$$-4\le 8y+10\le 10$$
$$-4-10\le 8y\le 10-10$$
$$-14\le 8y\le 0$$
$$-\frac{14}{8}\le y\le 0\quad,\quad -\frac{7}{4}\le y\le 0$$

The solution set is the interval $\left[-\frac{7}{4},0\right]$



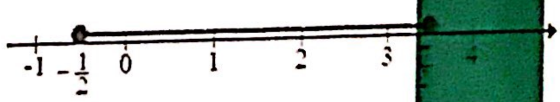
Exercise 14. $\sqrt{(3-2x)^2}\le 4$ $\boxed{\sqrt{(3-2x)^2}=|3-2x|}$

solution:

$$|3-2x|\le 4$$
$$|2x-3|\le 4$$
$$-4\le 2x-3\le 4$$
$$-4+3\le 2x\le 4+3$$
$$-1\le 2x\le 7$$
$$-\frac{1}{2}\le x\le\frac{7}{2}$$

$\boxed{|3-2x|=|2x-3|}$

The solution set is the interval $\left[-\frac{1}{2},\frac{7}{2}\right]$



Exercise 19. $-2|x|-2>4$

solution:

$-2|x|>4+2$

$-2|x|>6$

$|x|<-3$

يستحيل أن تكون القيمة المطلقة سالبة

The inequality has no solution

3. Absolute value with <u>more than</u> inequality (الحالة الثالثة : القيمة المطلقة في متباينة <u>أكبر من</u>)

$|f(x)|>a$ is equivalent $f(x)>a$ or $f(x)<-a$

Example : Solve the following inequalities and graph the solution set

1. $|2x+3|\geq 5$

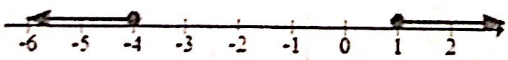
solution:

$2x+3\leq -5$ or $2x+3\geq 5$

$2x\leq -8$ or $2x\geq 2$ divide by 2

$x\leq -4$ or $x\geq 1$

The solution set is $(-\infty,-4]\cup[1,\infty)$



Exercise 15. $\sqrt{(2-7t)^2}>11$

solution:

$|2-7t|>11$

$|2-7t|=|7t-2|$

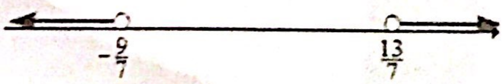
$|7t-2|>11$

$7t-2<-11$ or $7t-2>11$

$7t<-11+2$ or $7t>11+2$

$7t<-9$ or $7t>13$

$t<-\frac{9}{7}$ or $t>\frac{13}{7}$



The solution set is the interval $\left(-\infty,-\frac{9}{7}\right)\cup\left(\frac{13}{7},\infty\right)$

Exercise 20. $\frac{|x|}{5}+\frac{3}{2}\geq\frac{4}{9}$

solution:

$$\frac{|x|}{5}\geq\frac{4}{9}-\frac{3}{2}$$

$$\frac{|x|}{5}\geq-\frac{19}{18}$$ multiply by (5)

$$|x|\geq-\frac{95}{18}$$ القيمة المطلقة دائما موجبة . أي أن المتباينة متحققة لجميع قيم x

The solution set is the interval $(-\infty,\infty)$

هذه الجزئية موجودة بـ Example و لذلك تم اضافته لتكون المذكرة شاملة جميع أفكار المنهج

Definition : Modulus of Complex Number

Let $z=x+iy$ is a complex number

then the absolute value (or modulus) is $|z|=\sqrt{x^2+y^2}$

Ex : Let $z_1=4-3i$, $z_2=2+5i$, find

1. $|z_1|$ 2. $|z_1\bar{z}_2|$ 3. $|\mathrm{Re}(z_2)|$, $|\mathrm{Im}(z_1)|$ 4. $|3z_1+6i|$

solution:

1. $|z_1|=\sqrt{(4)^2+(-3)^2}=\sqrt{16+9}=5$

2. $|z_1\bar{z}_2|=|z_1||\bar{z}_2|=\sqrt{(4)^2+(-3)^2}\cdot\sqrt{(2)^2+(-5)^2}$
$=\sqrt{25}\cdot\sqrt{29}=5\sqrt{29}$

3. $|\mathrm{Re}(z_2)|=|5|=5$, $|\mathrm{Im}(z_1)|=|-3|=3$

4. $|3z_1+6i|=|3(4-3i)+6i|=|12-6i+6i|=|12|=12$

ION (2 – 4) : QUADRATIC EQUATIONS AND APPLICATIONS
المعادلات التربيعية و تطبيقاتها

adric Equation:

A quadratic equation in one variable in standard form $ax^2 + bx + c = 0$

Solving quadratic equation

1- By factoring
2- By completing square
3- By quadratic formula

حل المعادلة التربيعية
1- بالتحليل
2- اكمال المربع
3- القانون العام (الصيغة التربيعية)

rst : Solving a quadratic equation by factoring حل المعادلة التربيعية بطريقة التحليل

xample : Solve the following equations by factoring and check your answer
حل المعادلات التالية بالتحليل و تأكد من الحل

Exercise 1. $x^2 - 2x - 3 = 0$

solution:

$a = 1$

The factors of -3 are ± 1 , ± 3

The factors whose sum is -2 are 1 , -3

$$(x+1)(x-3) = 0$$

$$x+1=0 \quad \text{or} \quad x-3=0$$

$$x=-1 \quad \text{or} \quad x=3$$

The solution set is $\{-1,3\}$

* Check

$$(-1)^2 - 2(-1) - 3 = 0$$

$$(3)^2 - 2(3) - 3 = 0$$

اذا كان $a = 1$
نبحث عن العددين الذي حاصل ضربهم -3 و مجموعهم -2

2. $3x^2 - 15x = -18$

soution:

$3x^2 - 15x + 18 = 0$ divide all by 3

$x^2 - 5x + 6 = 0$

The factors of 6 are $\pm 1, \pm 2, \pm 3, \pm 6$

The factors whose sum is -5 are -2 and -3

$(x-2)(x-3) = 0$

$x - 2 = 0$ or $x - 3 = 0$

$x = 2$ or $x = 3$

The solution set is $\{2, 3\}$

* Check

$3(2)^2 - 15(2) + 18 = 12 - 30 + 18 = 0$

$3(3)^2 - 15(3) + 18 = 27 - 45 + 18 = 0$

Exercise 2. $2x^2 = 8x$

إذا كانت المعادلة التربيعية تحتوي على الحدين ax^2 and bx فقط (لا تحتوي الحد الثابت) : فإننا نحل المعادلة بأخذ x عامل مشترك

solution:

$2x^2 - 8x = 0$

$x(2x - 8) = 0$

$x = 0$ or $2x - 8 = 0$

$x = 0$ or $2x = 8$

$x = 0$ or $x = 4$

The solution set is $\{0, 4\}$

* Check

$2(0)^2 = 0 = 8(0)$

$2(4)^2 = 32 = 8(4)$

Exercise 3. $3w^2 + 13w = 0$

solution:

$$w(3w + 13) = 0$$

$w = 0$ or $3w + 13 = 0$

$w = 0$ or $3w = -13$

$w = 0$ or $w = -\frac{13}{3}$

The solution set is $\left\{-\frac{13}{3}, 0\right\}$

* Check

$3(0)^2 + 13(0) = 0$

$3(-\frac{13}{3})^2 + 13(-\frac{13}{3}) = 0$

اذا كان $b = 0$: نضع العدد الثابت باليمين و نأخذ الجذر التربيعي للطرفين

Exercise 4. $m^2 - 25 = 0$

solution:

$$(m - 5)(m + 5) = 0$$

$m - 5 = 0$ or $m + 5 = 0$

$m = 5$ or $m = -5$

The solution set is $\{-5, 5\}$

* Check

$(-5)^2 - 25 = 0$

$(5)^2 - 25 = 0$

4. $m^2 - 25 = 0$

solution:

$m^2 = 25$

$m = \pm\sqrt{25} = \pm 5$

The solution set is $\{-5, 5\}$

* Check

$(-5)^2 - 25 = 0$

$(5)^2 - 25 = 0$

Example (3)

3. $3x^2 + 12 = 0$

solution:

$3x^2 = -12$

$x^2 = -4$

$x = \pm\sqrt{-4} = \pm 2i$

The solution is $\{-2i, 2i\}$

Related Problem (3)

3. $\left(x+\frac{1}{3}\right)^2=9$

solution:

$$\sqrt{\left(x+\frac{1}{3}\right)^2}=\sqrt{9}$$ بأخذ الجذر للطرفين

$$\left|x+\frac{1}{3}\right|=3$$

$$x+\frac{1}{3}=\pm 3$$

$$x+\frac{1}{3}=3 \quad \text{or} \quad x+\frac{1}{3}=-3$$

$$x=3-\frac{1}{3} \quad \text{or} \quad x=-3-\frac{1}{3}$$

$$x=\frac{8}{3} \quad \text{or} \quad x=-\frac{10}{3}$$

The solution set is $\left\{-\frac{10}{3},\frac{8}{3}\right\}$

** Second: Solving Quadratic Equation using Completing the square إكمال المربع

Example: Solve the following equations by completing the square

1. $x^2+4x-3=0$

solution:

$$x^2+4x=3$$

$$x^2+4x+(2)^2=3+(2)^2$$

$$(x+2)^2=7$$

$$x+2=\pm\sqrt{7}$$

$$x=-2\pm\sqrt{7}$$

$$x=-2+\sqrt{7} \quad \text{or} \quad x=-2-\sqrt{7}$$

The solution set is $\left\{-2-\sqrt{7}\ ,\ -2+\sqrt{7}\right\}$

خطوات الحل

* جعل المجهول (x^2, x) بطرف والثابت 3 بالطرف الاخر
* بإضافة تربيع نصف معامل x لطرفي المعادلة
* بتحويل الطرف الايسر الى مربع كامل (انظر الطريقة)

$$x^2+4x+2^2=$$

$$(\ x \quad + \quad 2\ )^2$$

4) نكمل الحل بطريقة square root

Exercise 7. $16x^2+9=24x$

لحل بطريقة اكمال مربع يجب أن يكون معامل x^2 يساوي واحد

solution:

$$16x^2-24x=-9 \qquad \text{divide all by (16)}$$

$$x^2-\frac{3}{2}x=-\frac{9}{16}$$

$$x^2-\frac{3}{2}x+\left(\frac{3}{4}\right)^2=-\frac{9}{16}+\left(\frac{3}{4}\right)^2$$

$$\left(x-\frac{3}{4}\right)^2=0$$

$$x-\frac{3}{4}=0$$

$$x=\frac{3}{4}$$

The solution set is $\left\{\frac{3}{4}\right\}$

Exercise 8. $3z^2-8z+1=0$

solution: divide all by (3)

$$3z^2-8z=-1$$

$$z^2-\frac{8}{3}z=-\frac{1}{3}$$

$$z^2-\frac{8}{3}z+\left(\frac{4}{3}\right)^2=-\frac{1}{3}+\left(\frac{4}{3}\right)^2$$

$$\left(z-\frac{4}{3}\right)^2=\frac{13}{9}$$

$$z-\frac{4}{3}=\pm\sqrt{\frac{13}{9}}=\pm\frac{\sqrt{13}}{3}$$

$$z=\frac{4}{3}\pm\frac{\sqrt{13}}{3}$$

The solution set is $\left\{\frac{4}{3}-\frac{\sqrt{13}}{3},\frac{4}{3}+\frac{\sqrt{13}}{3}\right\}$

**Third: Solving Quadratic Equation by Quadratic Formula الصيغة العامة التربيعية

The equation $ax^2+bx+c=0$ has a solutions by quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

** Discriminant $\Delta=b^2-4ac$

* If $\Delta>0$ the equation has two distinct real solutions حلين مختلفين حقيقيين
* If $\Delta=0$ the equation has only one real solution repeated حل وحيد حقيقي مكرر
* If $\Delta<0$ the equation has two conjugate complex solutions حلين مركبين مترافقين

Example: Solve the following equations by quadratic formula

Exercise 10. $x^2-4x-1=0$

solution

$a=1$, $b=-4$, $c=-1$

$\Delta=b^2-4ac=(-4)^2-4(1)(-1)=20>0$ (Two distinct real solutions)

$$*\ x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-4)\pm\sqrt{20}}{2(1)}$$

$$=\frac{4\pm2\sqrt{5}}{2}$$

$$=\frac{4}{2}\pm\frac{2\sqrt{5}}{2}\quad=2\pm\sqrt{5}$$

The solution set is $\{2-\sqrt{5},2+\sqrt{5}\}$

Exercise 12. $2x^2+10x+11=0$

solution:

$a=2$, $b=10$, $c=11$

$\Delta=b^2-4ac=(10)^2-4(2)(11)=12>0$ (Two distinct real solutions)

$$*\ x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-10\pm\sqrt{12}}{2(2)}$$

$$=\frac{-10\pm2\sqrt{3}}{4}$$

$$=-\frac{10}{4}\pm\frac{2\sqrt{3}}{4}=-\frac{5}{2}\pm\frac{\sqrt{3}}{2}$$

The solution set is $\left\{-\frac{5}{2}-\frac{\sqrt{3}}{2},-\frac{5}{2}+\frac{\sqrt{3}}{2}\right\}$

Related Problem (5)

1. $3y^2 - 6y + 3 = 0$

solution:

$a = 3 \quad , \quad b = -6 \quad , \quad c = 3$

$\Delta = \sqrt{b^2 - 4ac} = \sqrt{(-6)^2 - 4(3)(3)} = 0$ (has only one real solution repeated)

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{0}}{2(3)} = \frac{6}{6} = 1$$

The solution set is $\{1\}$

Exercise 14. $4u^2 + 8u + 15 = 0$

solution: (Solving by quadratic formula)

$a = 4 \quad , \quad b = 8 \quad , \quad c = 15$

$\Delta = b^2 - 4ac = (8)^2 - 4(4)(15) = -176 < 0$ (Two conjugate complex solutions)

$$* \ u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-8 \pm \sqrt{-176}}{2(4)}$$

$$= \frac{-8 \pm 4\sqrt{11}i}{8}$$

$$= -1 \pm \frac{\sqrt{11}}{2}i$$

The solution set is $\left\{-1 - \frac{\sqrt{11}}{2}i, -1 + \frac{\sqrt{11}}{2}i\right\}$

Example : Solve the following equations by any method

حل المعادلات التالية بأي طريقة

Exercise 17. $1 + \frac{8}{x^2} = \frac{4}{x} \quad , \quad x \neq 0$

solution: multiply all by x^2

$$x^2 \cdot 1 + x^2 \cdot \frac{8}{x^2} = x^2 \cdot \frac{4}{x}$$

$$x^2 + 8 = 4x$$

$$x^2 - 4x = -8 \qquad \text{(by completing square)}$$

$$x^2 - 4x + (2)^2 = (2)^2 - 8$$

$$(x - 2)^2 = -4$$

$$x - 2 = \pm\sqrt{-4}$$

$$x = 2 \pm 2i$$

The solution set is $\{2 - 2i, 2 + 2i\}$

Exercise 18. $\frac{24}{10+x}+1=\frac{24}{10-x}$, $x\neq\pm10$

solution:

$$\frac{24}{10+x}\cdot\frac{10-x}{10-x}+1\cdot\frac{(10-x)(10+x)}{(10-x)(10+x)}=\frac{24}{10-x}\cdot\frac{10+x}{10+x}$$

$$\frac{24(10-x)+(10-x)(10+x)}{(10-x)(10+x)}=\frac{24(10+x)}{(10-x)(10+x)}$$

$$240-24x+100-x^2=240+24x$$ بحذف المقام مع المقام و تساوي البسط مع البسط

$$-x^2-48x+100=0$$

$$x^2+48x-100=0$$

$$(x+50)(x-2)=0$$

$$x+50=0 \quad \text{or} \quad x-2=0$$

$$x=-50 \quad \text{or} \quad x=2$$

The solution set is $\{-50,2\}$

Exercise 19. $\frac{2}{x-2}=\frac{4}{x-3}-\frac{1}{x+1}$, $x\neq 2,3,-1$

solution:

$$\frac{2}{x-2}\cdot\frac{(x-3)(x+1)}{(x-3)(x+1)}=\frac{4}{x-3}\cdot\frac{(x-2)(x+1)}{(x-2)(x+1)}-\frac{1}{x+1}\cdot\frac{(x-3)(x-2)}{(x-3)(x-2)}$$

$$\frac{2(x^2+x-3x-3)}{(x-2)(x-3)(x+1)}=\frac{4(x^2+x-2x-2)-(x^2-2x-3x+6)}{(x-3)(x-2)(x+1)}$$

$$2(x^2-2x-3)=4(x^2-x-2)-(x^2-5x+6)$$

$$2x^2-4x-6=4x^2-4x-8-x^2+5x-6$$

$$2x^2-4x-6=3x^2+x-14$$

$$-x^2-5x+8=0$$

$$x^2+5x-8=0 \qquad (\text{ by quadratic formula })$$

$$a=1 \quad , \quad b=5 \quad , \quad c=-8$$

$$\Delta=b^2-4ac=(5)^2-4(1)(-8)=57>0 \quad (\text{ Two distinct real solutions })$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-5\pm\sqrt{57}}{2(1)}$$

$$=-\frac{5}{2}\pm\frac{\sqrt{57}}{2}$$

The solution set is $\left\{-\frac{5}{2}-\frac{\sqrt{57}}{2},-\frac{5}{2}+\frac{\sqrt{57}}{2}\right\}$

تم اضافة هذا التمرين لان فكرته موجودة بـ (Example 5)

Exercise 21. $|12+7x|=x^2$

solution:

$$7x+12=\pm x^2$$

$7x+12=x^2$ or $7x+12=-x^2$

$x^2-7x-12=0$ or $x^2+7x+12=0$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ or $(x+3)(x+4)=0$

$=\dfrac{7\pm\sqrt{(-7)^2-4(1)(-12)}}{2(1)}$ or $x+3=0$ or $x+4=0$

$=\dfrac{7\pm\sqrt{97}}{2}$ or $x=-3$ or $x=-4$

$=\dfrac{7}{2}\pm\dfrac{\sqrt{97}}{2}$

The solution set is $\left\{-3\ ,\ -4\ ,\ \dfrac{7}{2}-\dfrac{\sqrt{97}}{2}\ ,\ \dfrac{7}{2}+\dfrac{\sqrt{97}}{2}\right\}$

Applications

1. The sum of a number and its reciprocal is $\dfrac{10}{3}$. Find the numbers.

مجموع عدد و مقلوبه يساوي $\dfrac{10}{3}$. أوجد العددين

solution:

Let the number is x and its reciprocal is $\dfrac{1}{x}$

$$* \quad x+\frac{1}{x}=\frac{10}{3}$$

$$\frac{3x}{3x}\cdot\frac{x}{1}+\frac{3}{3}\cdot\frac{1}{x}=\frac{10}{3}\cdot\frac{x}{x}$$

$$\frac{3x^2}{3x}+\frac{3}{3x}=\frac{10x}{3x}$$

$$\frac{3x^2+3}{3x}=\frac{10x}{3x}$$

$$3x^2+3=10x$$

$3x^2-10x+3=0$ (By factoring or any method)

$(3x-1)(x-3)=0$

$3x-1=0$ or $x-3=0$

$3x=1$ or $x=3$

$x=\dfrac{1}{3}$ or $x=3$

The numbers are $\dfrac{1}{3}$ and 3

Exercise 24. Find the two numbers such that their sum is 21 and thier product is 104

أوجد عددين بحيث مجموعهما 21 و حاصل ضربهما 104

solution:

Let the numbers are x and y

$x + y = 21 \quad \Rightarrow \quad y = 21 - x$

* Thier product is 104 $\Rightarrow \quad x \cdot y = 104$

$$x(21 - x) = 104$$

$$21x - x^2 = 104$$

$$x^2 - 21x + 104 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-21) \pm \sqrt{(-21)^2 - 4(1)(104)}}{2(1)}$$

$$= \frac{21 \pm \sqrt{25}}{2} \qquad = \frac{21 \pm 5}{2}$$

$$x = \frac{21+5}{2} = 13 \qquad \text{or} \qquad x = \frac{21-5}{2} = 8$$

The two numbers are 8 and 13

Exercise 25. Find two consecutive positive even integers whose product is 168

أوجد عددين صحيحين زوجين متتالين حاصل ضربهما 168

solution:

Let the two consecutive positive even integers are x and $x + 2$

* Their product is 168 $\Rightarrow \qquad x(x + 2) = 168$

$$x^2 + 2x = 168$$

$$x^2 + 2x + (1)^2 = 168 + (1)^2$$

$$(x + 1)^2 = 169$$

$$x + 1 = \pm\sqrt{169} = \pm 13$$

$$x + 1 = 13 \qquad \text{or} \qquad x + 1 = -13$$

$$x = 12 \qquad \text{or} \qquad x = -14 \text{ (refused)}$$

The two consecutive positive even integers are 12 and 14

Example 5

Ex : The width of a rectangle is three centimeters less than the length . If the area of the rectangle is 54 cm^2 , find the dimensions of the rectangle.

solution:

عرض مستطيل أقل من طوله بـ 3 . اذا كان مساحة المستطيل 54 أوجد أبعاد المستطيل

Let the length $= x$, then the width $= x - 3$

* Area of rectangle = (length) · (width)   مساحة المستطيل = الطول × العرض

$$x(x-3) = 54$$

$$x^2 - 3x - 54 = 0$$

$$(x-9)(x+6) = 0$$

$$x = 9 \quad \text{or} \quad x = -6 \ \text{refused}$$

The length of the rectangle is 9 and the width is $= 9 - 3 = 6$

Example 9

Ex : A garden measuring 12 meters by 16 is to have a pedestrain pathway installed all around it , increasing the total area to 285 square meters . What will be the width of pathway.

حديقة قياس ابعادها 12 في 16 ، لتثبيت طريق مشاة حولها يزيد المساحة الكلية الى 285 . ما هو عرض الطريق ؟

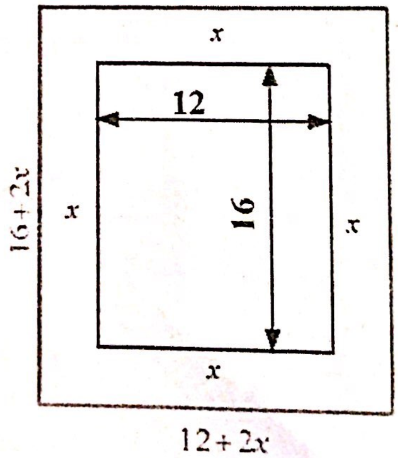
solution:

Let the width of the pathway is x

total width $= 12 + x + x = 12 + 2x$

total length $= 16 + x + x = 16 + 2x$

* Total new area = (total width) · (total length)



$$(12 + 2x)(16 + 2x) = 285$$

$$192 + 24x + 32x + 4x^2 = 285$$

$$4x^2 + 56x - 93 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-56 \pm \sqrt{(56)^2 - 4(4)(-93)}}{2(4)}$$

$$= \frac{-56 \pm 68}{8}$$

$$x = \frac{-56 - 68}{8} = -15.5 \ \text{refused} \quad \text{or} \quad x = \frac{-56 + 68}{8} = 1.5$$

The width of the pathway is 1.5

CHAPTER 3 : FUNCTIONS الدوال

SECTION (3 – 1) : FUNCTIONS

DEFINITION : FUNCTION

A function f from a set X to a set Y where each element x in X which assigns one and only one element y in Y

الدالة : كل عنصر في X يرتبط مع عنصر واحد فقط في Y

Example: Determine which of the following sets is a function . If it is a function , what is its domain and range ? Explain your reason for any that do not define a function

حدد أي من المجموعات التالية دالة . اذا كانت دالة أوجد مجالها و مداها مع ذكر السبب اذا كانت ليست دالة

Exercise 1. $f = \{(2,3),(3,3),(-2,3),(1,3),(0,3)\}$.

solution:

f is a function

لاحظ أن عناصر المركبة الأولى لم تتكرر

$D_f = \{2,3,-2,1,0\}$

Range: $R = \{3\}$

Exercise 2. $g = \{(5,1),(2,2),(-1.5\ ,\ 2),(5,3),(1,7)\}$

solution:

g is not a function

because the order pairs $(5,1)$ and $(5,3)$ have the same x – coordinate

ليست دالة لأن العنصر 5 له أكثر من صورة

Exercise 4. $k = \{(4,0),(4,-1),(4,4),(4,2),(4,3)\}$

solution:

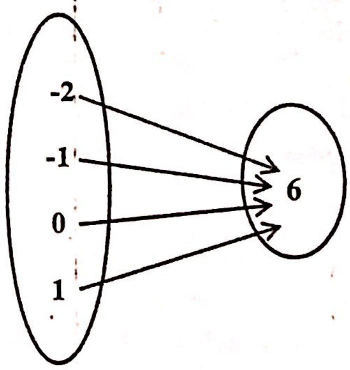
k is not a function

because all order pairs have the same x – coordinate

ليست دالة لأن جميع الأزواج المرتبة لها نفس احداثي x

Example , Determine which of the following diagram represent a function .
Explain your reason for any that do not define a function

Exercise 7.



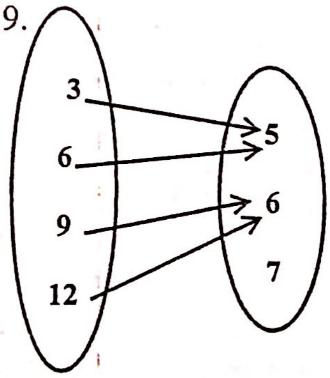
solution:

A function

Domain $= \{-2, -1, 0, 1\}$

Range $= \{6\}$

Exercise 9.



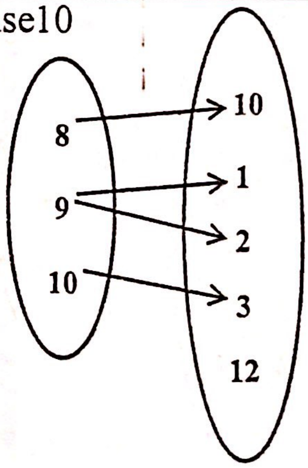
solution:

A function

Domain $= \{3, 6, 9, 12\}$

Range $= \{5, 6\}$

Exercise10



solution:

Not a function ,

because the order pairs (9,1) and (9,2)

have the same x – coordinate

Example , Determine which of the following define a function in terms of the independent variable x . Explain your reason for any that do not define a function.

طريقة الحل:

* نحل المعادلة في y (بمعنى جعل y بطرف وباقي حدود المعادلة بالطرف الأخر

*اذا ظهر الإشارة ± فهي ليست دالة (not function) لأن لكل قيمة لـ x ستكون لها قيمتان في y

*اذا لم يظهر الإشارة ± فانها دالة (function) لأن لكل قيمة لـ x ستظهر صورة واحدة في y

Exerrcise 11. $4x - 5y = 20$

solution:

$$-5y = -4x + 20$$

$$y = \frac{4}{5}x - 4$$

define a function , because for each value of x , it has exactly one value for y

Exercise 15. $x + y^2 = 10$

solution:

$$y^2 = 10 - x$$

$$y = \pm\sqrt{10 - x}$$

Does not a function , because the orderd pairs $(1,3)$ and $(1,-3)$ satisfying $x + y^2 = 10$ and have the same x – coordinate

Exercise 16. $xy - 4y = 1$

solution:

$$y(x - 4) = 1$$

$$y = \frac{1}{x - 4} \quad , \quad x \neq 4$$

define a function , because for each value of $x \neq 4$, it has exactly one value for y

Exercise 18. $x^2 + y^2 = 25$

solution:

$$y^2 = 25 - x^2$$

$$y = \pm\sqrt{25 - x^2}$$

Does not a function , because the orderd pairs $(0,5)$ and $(0,-5)$ satisfying $x^2 + y^2 = 25$ and have the same x – coordinate

Domain of The Functions مجال الدوال

First : Domain of Polynomial مجال كثيرة الحدود

The domain of polynomial always $(-\infty,\infty)$ or $\mathbb{R}$.

Example: find the domain of each function

Exercise 27. $f(x)=x^3-4x+1$

solution:

Domain $D_f=\mathbb{R}=(-\infty,\infty)$

Exercise 28. $f(x)=x^6-\sqrt{2}x^3-7x+5$

solution:

Domain $D_f=\mathbb{R}=(-\infty,\infty)$

Second : Domain of Fraction مجال الدالة الكسرية

The domain of Fration is $(-\infty,\infty)-\{\text{zero of denominator}\}$

مجال الدالة الكسرية : جميع الأعداد الحقيقية ماعدا أصفار المقام

Example: find the domain of each function

Exercise 39. $g(w)=\dfrac{2}{w-1}$

solution:

$w-1\neq 0$, $w\neq 1$

Domain $D_g=\{w\in\mathbb{R}:w\neq 1\}=(-\infty,1)\cup(1,\infty)$

Exercise 41. $g(w) = \dfrac{w-1}{w^2 - w - 6}$

solution:

$$w^2 - w - 6 \neq 0$$

$$(w-3)(w+2) \neq 0$$

$$w \neq 3 \;, \quad w \neq -2$$

Domain $D_g = \{w \in \mathbb{R} : w \neq -2, 3\} = (-\infty, -2) \cup (-2, 3) \cup (3, \infty)$

Third : Domain of Square Root مجال الجذر التربيعي

The domain of even root is (Negative not allowed under even root)

مجال الدالة الجذرية : ماداخل الجذر ≤ صفر . اذا كان الجذر بالبسط

ماداخل الجذر > صفر . اذا كان الجذر بالمقام

** هام جدا : مجال الجذر الفردي هو الأعداد الحقيقية

Example: find the domain of each function

Exercise 32. $f(x) = \sqrt{1-7x}$

solution :

$$1 - 7x \geq 0$$

$$-7x \geq -1$$

$$x \leq \frac{1}{7}$$

عند الضرب أو القسمة بعدد سالب نغير علامة المتباينة

$$D_f = \left\{x \in \mathbb{R} : x \leq \frac{1}{7}\right\} = \left(-\infty, \frac{1}{7}\right]$$

Exercise 33. $f(x) = \dfrac{1}{\sqrt{x-5}}$

solution:

$$x - 5 > 0$$

$$x > 5$$

$$D_f = \{x \in \mathbb{R} : x > 5\} = (5, \infty)$$

Exercise 38. $f(t)=\sqrt[3]{1-t^2}$

solution: مجال الجذر الفردي هو جميع الأعداد الحقيقية

Domain $D_f=(-\infty,\infty)$

- نأخذ أمثلة ايجاد دوال تحتوي على دمج دوال جذر و كسر ....

Example: find the domain of each function

Example 6 (مثله)

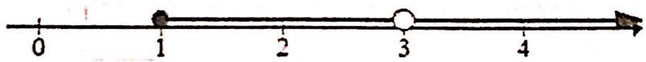
1. $f(x)=\dfrac{\sqrt{x-1}}{x-3}$

نحصل على مجال الجذر و نحذف منها أصفار المقام

solution:

Domain of $\sqrt{x-1}$: $x-1\geq 0$, $x\geq 1$

and non-zero of denominator: $x-3\neq 0$, $x\neq 3$



Domain $D_f=\{x\in\mathbb{R}: x\geq 1 \text{ and } x\neq 3\}=[1,3)\cup(3,\infty)$

Exercise 36. $f(x)=\dfrac{\sqrt{x}+4x}{x^3-x}$

نحصل على مجال الجذر و نحذف منها أصفار المقام

solution:

Domain of $\sqrt{x}$: $x\geq 0$

and non-zero of denominator: $x^3-x\neq 0$, $x(x^2-1)\neq 0$

$x(x-1)(x+1)\neq 0$

$x\neq 0$, $x\neq 1$ and $x\neq -1$

Domain $D_f=\{x\in\mathbb{R}: x>0 \text{ and } x\neq 1\}=(0,1)\cup(1,\infty)$

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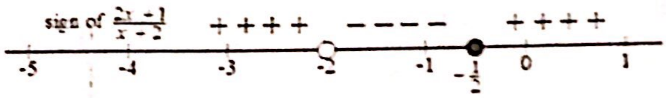
نحصل أصفار البسط و المقام و نبحث اشارة الدالة و يكون
مجاله هي الفترات الموجبة
لاحظ أصفار البسط مقبولة في المجال وأصفار المقام مرفوضة

Exercise 35. $f(x) = \sqrt{\dfrac{2x+1}{x+2}}$

solution:

The domain of the function f is the solution of the inequality $\dfrac{2x+1}{x+2} \geq 0$

The zeros are $2x+1=0$ and $x+2=0 \Rightarrow x = -\frac{1}{2}$ and $x=-2$



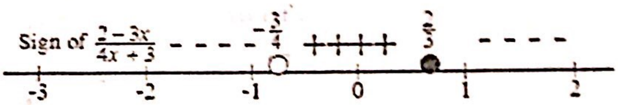
Domain $D_f = \left\{x \in \mathbb{R} : x < -2 \text{ or } x \geq -\frac{1}{2}\right\} = (-\infty, -2) \cup \left[-\frac{1}{2}, \infty\right)$

Related Problem 5. $f(x) = \sqrt{\dfrac{2-3x}{4x+3}}$

solution:

The domain of the function f is the solution of the inequality $\dfrac{2-3x}{4x+3} \geq 0$

The zeros are $2-3x=0$ and $4x+3=0 \Rightarrow x = \frac{2}{3}$ and $x = -\frac{3}{4}$



Domain $D_f = \left\{x \in \mathbb{R} : -\frac{3}{4} < x \leq \frac{2}{3}\right\} = \left(-\frac{3}{4}, \frac{2}{3}\right]$

Exercises:

Exercise 19: Let $f(x) = \sqrt{3}$ find

1. $f(-2)$ 2. $f(\sqrt{5})$

solution:

1. $f(-2) = \sqrt{3}$
2. $f(\sqrt{5}) = \sqrt{3}$

الدالة الثابتة مجالها الأعداد الحقيقية

Exercise 22: Let $g(t)=|2-t|$ find

1. Domain of g 2. $f(2)$

solution:

1. Domain of $g(x)$ is $\mathbb{R}$

because g is absolute value function

2. $g(2)=|2-2|=0$

Exercise 25: Let $g(t)=\sqrt[3]{t}$ find

1. $f(27)$ 2. $f\left(\frac{1}{8}\right)$

solution:

1. $g(27)=\sqrt[3]{27}=3$
2. $g\left(\frac{1}{8}\right)=\sqrt[3]{\frac{1}{8}}=\frac{1}{2}$

Exercise 26: $g(x)=\dfrac{3x^2-4x-1}{2x^2+5x-3}$, find $g(-1)$

solution:

* $g(-1)=\dfrac{3(-1)^2-4(-1)-1}{2(-1)^2+5(-1)-3}=-1$

Exercise 42: Determine which of the following define a function . Explain your reason for any that do not define a function

a. The domain consists of the number 2 , which is assigned the number 4

solution

is a function $\{(2,4)\}$ is a function

c. $f(x)=\pm\sqrt{x}$

solution:

f is not a function since the ordered pairs $(1,1)$ and $(1,-1)$ have the same x – coordinate

ليست دالة لأن مثلا العنصر 1 له أكثر من صورة

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g. $g(x) = \begin{cases} x-1 & \text{for } x < 0 \\ 12x-6 & \text{for } x > 0 \end{cases}$

solution:

g is a function

h. $g(x) = \begin{cases} 2-3x^2 & \text{for } x \le 1 \\ 3x^4-3 & \text{for } x \ge 1 \end{cases}$

solution:

لاحظ أن المعادلة معرفة عند 1 في الطرفين
أي ان العنصر 1 مرتبط بأكثر من عنصر

$g(1) = 2-3x^2 = 2-3(1)^2 = -1$

$g(1) = 3x^4 - 3 = 3(1)^4 - 3 = 0$

g is not a function since the ordered pairs $(1,-1)$ and $(1,0)$ have the same x – coordinate

Equality of Function : تساوي الدوال

The two function are equal if they have the same domain and the same value for each number in their domain

الدالتين متساويتان اذا كان لهما نفس المجال . و صورة كل عدد في الدالتين له نفس القيمة

Exercise 43: Determine whether f and g are the same of the following

b. $f(x) = \sqrt{x^2}$ and $g(x) = |x|$

solution:

$f(x) = \sqrt{x^2} = |x| = g(x)$

and have the same domain $\mathbb{R}$

Then f and g are the same

c. $f(x) = \sqrt{x}$ for $x \ge 0$; $g(x) = \sqrt{x}$

solution:

f and g are the same because they have the same domain $x \ge 0$

67

d. $f(x)=\frac{x^3-4x}{x^3-4x}$; $g(x)=1$

solution:

$x^3-4x=0$, $x(x^2-4)=0 \Rightarrow x=0$, $x=\pm 2$

$f(x)=\frac{x^3-4x}{x^3-4x}=1$, domain of f is $\mathbb{R}-\{-2,0,2\}$

but domain of $g(x)=1$ is $\mathbb{R}$

Then f and g are not the same

---

e. $f(x)=\frac{x-1}{x^2-1}$; $g(x)=\frac{1}{x+1}$

solution:

$f(x)=\frac{x-1}{x^2-1}=\frac{x-1}{(x-1)(x+1)}=\frac{1}{x+1}$ Domain of f is $\mathbb{R}-\{-1,1\}$

but Domain of $g(x)=\frac{1}{x+1}$ is $\mathbb{R}-\{-1\}$

Then f and g are not the same

---

f. $f(x)=\frac{x^2-5x+6}{x-2}$; $g(x)=x-3$ for $x\neq 2$

solution:

$f(x)=\frac{x^2-5x+6}{x-2}=\frac{(x-3)(x-2)}{x-2}=x-3$

f and g are the same

راح ارسلها صوره اوضح .. اسفه 😶💔

Exercise 45: Find a formula for that assigns to each x greater than -1 the number obtained by squaring x, then subtracting $2x$, and finally adding $\sqrt{2}$

solution:

$f(x)=x^2-2x+\sqrt{2}$ for $x>-1$

Exercise 46: Find a formula for the function A that expresses the area of a circle in terms of the radius

solution:

$A(r)=\pi r^2$

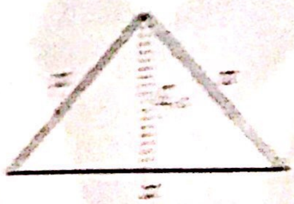
Ex: Find a formula for the function A that expresses the area of an equilateral triangle in terms of the length of an edge

solution:

Let the length of an edge is x

* Area of triangle $=\frac{1}{2}(\text{base})(\text{height})$

$$=\frac{1}{2}x\left(\frac{\sqrt{3}}{2}x\right)=\frac{\sqrt{3}}{4}x^2$$



ction (3 – 2) : Polynomials and Rational Functions

دوال كثيرات الحدود و الدوال الكسرية

xercises 1 – 7 , Determine which of the following define a polynomial function . Explain your reason for any that do not define a polynomial function

حدد أي من الدوال التالية تعرف كثيرة حدود واشرح السبب اذا كانت ليست كثيرة حدود

Exercise 1. $f(x) = 3x^{-4} + 2x^{15} + x^{-2} + 13$

solution:

Not a polynomial , since its first and third has a negative power

ليست كثيرة حدود لأن الحد الأول و الثالث لها أس سالب

Exercise 2. $g(x) = 5x^2 - x^3 + x^7$

solution:

Polynomial of degree 7 and leading coefficient 1

كثيرة حدود الدرجة 7 و معاملها الرئيسي 1

Exercise 4. $k(x) = 5$

solution:

Polynomial of degree 0 and leading coffiecient 5

Exercise 5. $f(x) = \dfrac{x^5 - 3x + 2}{3x^{-2} - 6x}$

solution:

Not a polynomial , since it's not in the form $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

Exercise 6. $g(x) = \dfrac{4x^6 + x^3}{x}$

solution:

Not a polynomial , since it's not in the form $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

هام جدا : (تذكر)

1- مجال كثيرة الحدود : هي جميع الأعداد الحقيقية

2- مجال الدالة الكسرية : جميع الأعداد الحقيقية ماعدا أصفار المقام

Exercise 8 – 17 , Find the domain of the following functions.

Exercise 8. $f(x) = 3x^3 - 3x^2 + 4$

solution:

Domain $D_f = \mathbb{R} = (-\infty, \infty)$

Exercise 12. $f(x) = \dfrac{x-1}{x^2+x+1}$

solution:

$x^2 + x + 1 = 0$ has no real solution

Domain D_f is $(-\infty, \infty)$

Exercise 15. $f(x) = \dfrac{3x^2 - x + 4}{\sqrt{x} - 2}$

نحصل على مجال الجذر و نرفض أصفار المقام

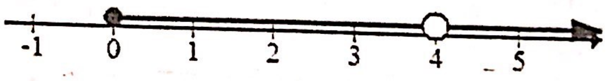
solution:

The domain of $\sqrt{x}$ is : $x \ge 0$

The zeros of denominator : $\sqrt{x} - 2 = 0$

$\sqrt{x} = 2$

$x = 4$



Domain D_f is : $[0,4) \cup (4, \infty)$

Exercise 16. $f(x) = \dfrac{3x^2 - x + 4}{\sqrt{2x-4} - 3}$

نحصل على مجال الجذر و نرفض أصفار المقام

solution:

The domain of $\sqrt{2x-4}$ is : $2x - 4 \ge 0$, $2x \ge 4$, $x \ge 2$

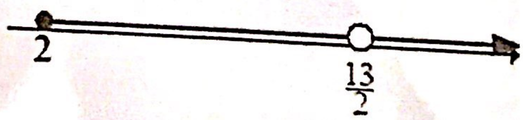
The zeros of denominator : $\sqrt{2x-4} - 3 = 0$

$\sqrt{2x-4} = 3$

$2x - 4 = 9$

$2x = 13$, $x = \dfrac{13}{2}$

Domain D_f is : $\left[2, \dfrac{13}{2}\right) \cup \left(\dfrac{13}{2}, \infty\right)$



Exercise 18 – 31 : Divide

Exercise 18. $\dfrac{20x^4 + x^3 + 2x^2}{x^3}$

solution:

$$\begin{array}{r|l} & 20x + 1 \\ \hline x^3 & 20x^4 + x^3 + 2x^2 \\ & \underline{{}^-20x^4} \\ & \qquad\quad x^3 + 2x^2 \\ & \qquad\quad \underline{{}^-x^3} \\ & \qquad\qquad\quad 2x^2 \end{array}$$

$*\ \dfrac{20x^4 + x^3 + 2x^2}{x^3} = 20x + 1 + \dfrac{2x^2}{x^3}$

$= 20x + 1 + \dfrac{2}{x}$

Exercise 22. $\dfrac{4y^3 - 2y^2 - 3}{2y^2 - 1}$

solution:

$$\begin{array}{r|l} & 2y - 1 \\ \hline 2y^2 - 1 & 4y^3 - 2y^2 \qquad - 3 \\ & \underline{{}^-4y^3 \qquad \overset{+}{-}2y} \\ & \qquad -2y^2 + 2y - 3 \\ & \qquad \underline{\overset{+}{-}2y^2 \qquad \overset{-}{+}1} \\ & \qquad\qquad 2y - 4 \end{array}$$

$*\ \dfrac{4y^3 - 2y^2 - 3}{2y^2 - 1} = 2y - 1 + \dfrac{2y - 4}{2y^2 - 1}$

Exercise 23. $\frac{x^2+13x+32}{x+5}$

solution:

$$
\begin{array}{r}
x+8 \\
x+5\overline{\smash{)}\,x^2+13x+32} \\
\underline{{}^{-}x^2\overset{-}{+}5x} \\
8x+32 \\
\underline{{}^{-}8x\overset{-}{+}40} \\
-8
\end{array}
$$

* $\frac{x^2+13x+32}{x+5}=x+8-\frac{8}{x+5}$

Exercise 25. $\frac{x^2-4x-38}{x-8}$

solution:

$$
\begin{array}{r}
x+4 \\
x-8\overline{\smash{)}\,x^2-4x-38} \\
\underline{{}^{-}x^2\overset{+}{-}8x} \\
4x-38 \\
\underline{{}^{-}4x\overset{+}{-}32} \\
-6
\end{array}
$$

* $\frac{x^2-4x-38}{x-8}=x+4-\frac{6}{x-8}$

Exercise 28. $\frac{2u^3-4u^2+7u+7}{u^2+2u-1}$

solution:

$$
\begin{array}{r}
2u-8 \\
u^2+2u-1\overline{\smash{)}\,2u^3-4u^2+7u+7} \\
\underline{{}^{-}2u^3\overset{-}{+}4u^2\overset{+}{-}2u} \\
-8u^2+9u+7 \\
\underline{{}^{+}-8u^2\overset{+}{-}16u\overset{-}{+}8} \\
25u-1
\end{array}
$$

* $\frac{2u^3-4u^2+7u+7}{u^2+2u-1}=2u-8+\frac{25u-1}{u^2+2u-1}$

Example . Use long division to find the quotient Q(x) and remainder R(x) the each rational function

Exercise 32. $\dfrac{x^3+15x^2+49x-55}{x+7}$

solution:

$$
\begin{array}{r|l}
 & x^2+8x-7 \\
\hline
x+7 & x^3+15x^2+49x-55 \\
 & \underline{\overset{-}{x^3}\overset{-}{+}7x^2} \\
 & \quad 8x^2+49x-55 \\
 & \quad \underline{\overset{-}{8x^2}\overset{-}{+}56x} \\
 & \qquad -7x-55 \\
 & \qquad \underline{\overset{+}{-7x}\overset{+}{-}49} \\
 & \qquad\qquad -6
\end{array}
$$

$Q(x)=x^2+8x-7$, $R(x)=-6$

* $\dfrac{x^3+15x^2+49x-55}{x+7}=x^2+8x-7-\dfrac{6}{x+7}$

Exercise 35. $\dfrac{x^3-46x+22}{x+7}$

solution:

$$
\begin{array}{r|l}
 & x^2-7x+3 \\
\hline
x+7 & x^3 \qquad -46x+22 \\
 & \underline{\overset{-}{x^3}\overset{-}{+}7x^2} \\
 & \quad -7x^2-46x+22 \\
 & \quad \underline{\overset{+}{-7x^2}\overset{+}{-}49x} \\
 & \qquad 3x+22 \\
 & \qquad \underline{\overset{-}{3x}\overset{-}{+}21} \\
 & \qquad\qquad 1
\end{array}
$$

$Q(x)=x^2-7x+3$, $R(x)=1$

* $\dfrac{x^3-46x+22}{x+7}=x^2-7x+3+\dfrac{1}{x+7}$

Exercise 36. $\frac{x^6+2x^4+6x-9}{x^3+3}$

solution:

$$\begin{array}{r|l} & x^3+2x-3 \\ \hline x^3+3 & x^6+2x^4\qquad +6x-9 \\ & \underline{\mp x^6 \qquad \mp 3x^3} \\ & 2x^4-3x^3+6x-9 \\ & \underline{\mp 2x^4 \qquad \mp 6x} \\ & -3x^3 \qquad -9 \\ & \underline{\pm 3x^3 \qquad \pm 9} \\ & 0 \end{array}$$

$Q(x)=x^3+2x-3$, $R(x)=0$

* $\frac{x^6+2x^4+6x-9}{x^3+3}=x^3+2x-3$

Example , write each rational function on the form

$$q(x)=Q(x)+\frac{R(x)}{g(x)}$$

Exercise 39. $\frac{4x^3-21x^2+6x+19}{4x+3}$

solution:

$$\begin{array}{r|l} & x^2-6x+6 \\ \hline 4x+3 & 4x^3-21x^2+6x+19 \\ & \underline{\mp 4x^3 \mp 3x^2} \\ & -24x^2+6x+19 \\ & \underline{\pm 24x^2 \pm 18x} \\ & 24x+19 \\ & \underline{\mp 24x \mp 18} \\ & 1 \end{array}$$

* $\frac{4x^3-21x^2+6x+19}{4x+3}=x^2-6x+6+\frac{1}{4x+3}$

Exercise 40. $\dfrac{3x^4+9x^3-5x^2-6x+2}{3x^2-2}$

solution:

$$
\begin{array}{r|l}
 & x^2+3x-1 \\
\hline
3x^2-2 & 3x^4+9x^3-5x^2-6x+2 \\
 & \underline{\overset{-}{3x^4} \qquad \overset{+}{-}2x^2} \\
 & 9x^3-3x^2-6x+2 \\
 & \underline{\overset{-}{9x^3} \qquad \overset{+}{-}6x} \\
 & -3x^2 \qquad +2 \\
 & \underline{\overset{+}{-}3x^2 \qquad \overset{-}{+}2} \\
 & 0 \qquad 0
\end{array}
$$

* $\dfrac{3x^4+9x^3-5x^2-6x+2}{3x^2-2}=x^2+3x-1$

Exercise 41. $\dfrac{x^4-13x-42}{x^2-x+5}$

solution:

$$
\begin{array}{r|l}
 & x^2+x-4 \\
\hline
x^2-x+5 & x^4 \qquad -13x-42 \\
 & \underline{\overset{-}{x^4}\overset{+}{-}x^3\overset{-}{+}5x^2} \\
 & x^3-5x^2-13x-42 \\
 & \underline{\overset{-}{x^3}\overset{+}{-}x^2\overset{-}{+}5x} \\
 & -4x^2-18x-42 \\
 & \underline{\overset{+}{-}4x^2\overset{-}{+}4x\overset{+}{-}20} \\
 & -22x-22
\end{array}
$$

* $\dfrac{x^4-13x-42}{x^2-x+5}=x^2+x-4+\dfrac{-22x-22}{x^2-x+5}$

SECTION (3.3) : GRAPHS الرسومات

First : Graph of Linear Functions رسم الدوال الخطية

* $f(x) = mx + b$
* $f(x) =$ constant

Exercise 1 – 23 : Sketch the graph of the function.

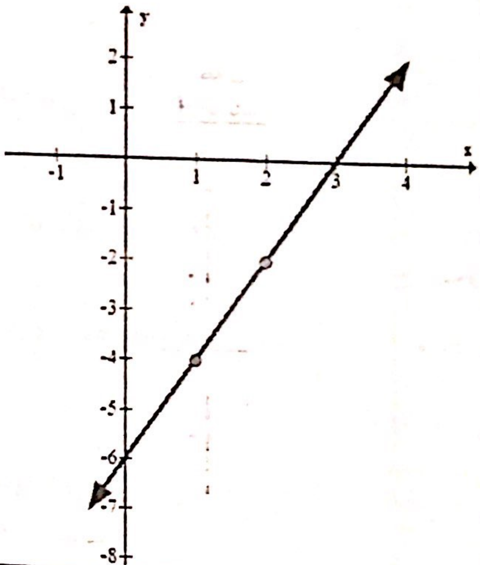
Exercise 1. $f(x) = 2x - 6$

solution

$y = 2x - 6$ is a line with slope $= 2$ and y intercept -6

لرسم دالة من الدرجة الأولى نحدد نقطتين ثم نوصلهم

x	1	2
y	-4	-2



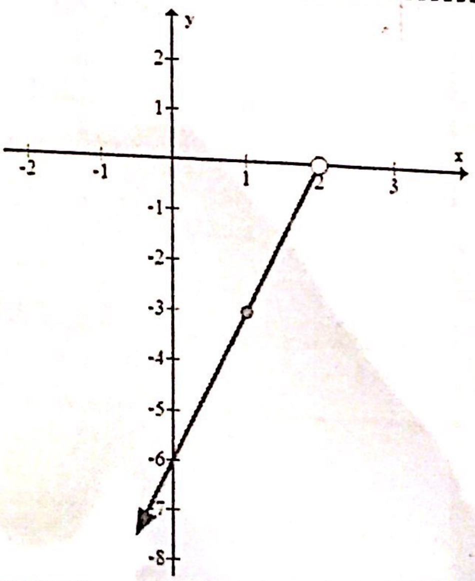
Exercise 2. $f(x) = 3x - 6 \quad , \quad x < 2$

solution:

$y = 3x - 6$ is a line with slope $= 3$ with y intercept -6

x	1	2 (open point)
y	-3	0

لاحظ ان الدالة معرفة عن القيم أقل من 2



Exercise 4. $h(x)=-3$

$h(x)=-3$ تمثل دالة ثابتة توازي محور x

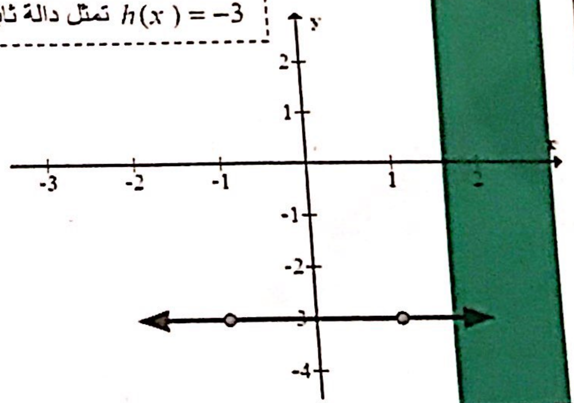
solution:

$h(x)=-3$ is a constant function

is a line parallel x – axis

y intercept -3

x	-1	1
y	-3	-3



Second : Graph of Quadratic Function رسم الدوال التربيعية

$$* \; f(x)=ax^2+bx+c$$

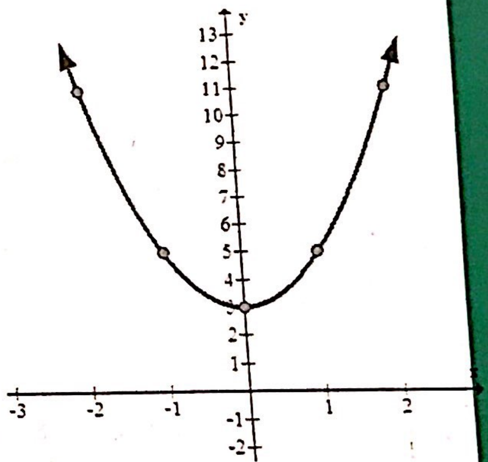
نعوض في الدالة التربيعية بعدة نقاط ثم نصلها (كلما زادت النقاط زادت دقة الرسمة)

Exercise 5. $f(x)=2x^2+3$

solution:

$y=2x^2+3$ quadratic equation

x	-2	-1	0	1	2
y	11	5	3	5	11

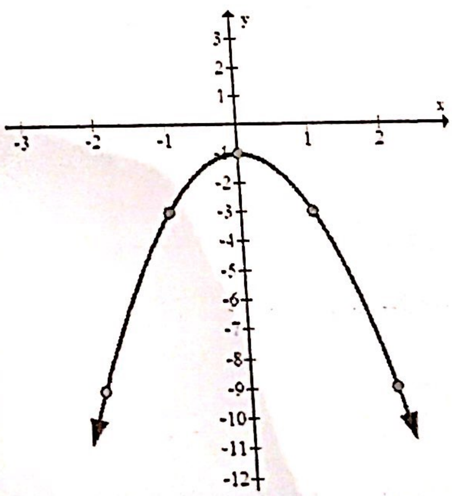


2. $g(x)=-2x^2-1$

solution:

$y=-2x^2-1$

x	-2	-1	0	1	2
y	-9	-3	-1	-3	-9

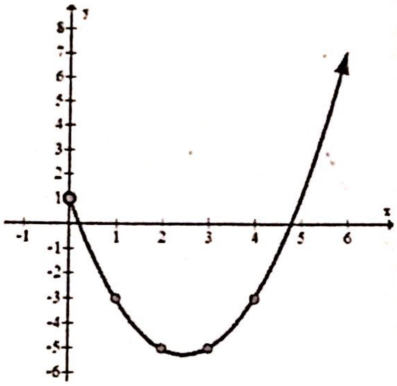


Exercise 7. $f(x)=x^2-5x+1 \quad , \quad x \geq 0$

solution:

$y=x^2-5x+1$ quadratic function

x	0	1	2	3	4
y	1	-3	-5	-5	-3



Third : Graph of Absolute Value Function رسم دوال القيمة المطلقة

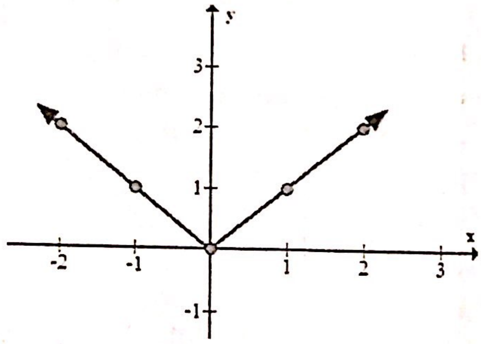
لرسم دالة القيمة المطلقة نعيد تعريف الدالة ثم نعوض عن عدة نقاط

Example 3. Sketch the graph of $f(x)=|x|$

solution:

$$f(x)=|x|=\begin{cases} x & , \ x \geq 0 \\ -x & , \ x<0 \end{cases}$$

x	-2	-1	0	1	2
y	2	1	0	1	2



Related Problem (3)

Ex : Let $f(x)=3|x|-2$. Sketch the graph of f

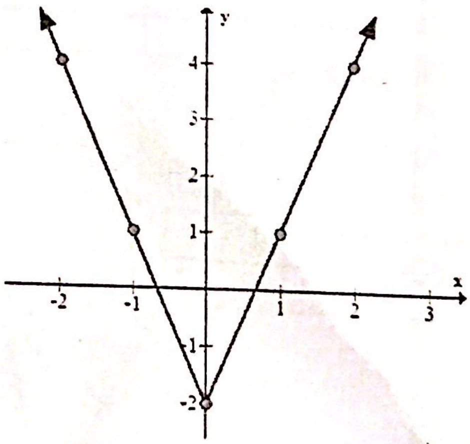
solution:

$$|x|=\begin{cases} x & , \ x \geq 0 \\ -x & , \ x<0 \end{cases}$$

$$3|x|=\begin{cases} 3x & , \ x \geq 0 \\ -3x & , \ x<0 \end{cases}$$

$$f(x)=3|x|-2=\begin{cases} 3x-2 & , \ x \geq 0 \\ -3x-2 & , \ x<0 \end{cases}$$

x	-2	-1	0	1	2
y	4	1	-2	1	4



Exercise 9. $g(x) = -2|x+1|+2$

solution:

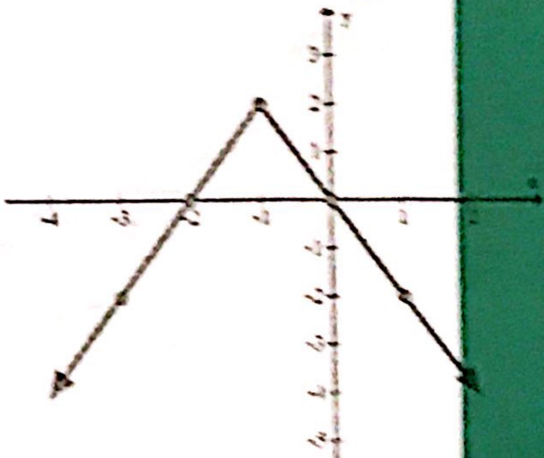
$$|x+1| = \begin{cases} x+1 & , x \geq -1 \\ -(x+1) & , x < -1 \end{cases}$$

$$-2|x+1| = \begin{cases} -2(x+1) & , x \geq -1 \\ 2(x+1) & , x < -1 \end{cases}$$

$$-2|x+1|+2 = \begin{cases} -2(x+1)+2 & , x \geq -1 \\ 2(x+1)+2 & , x < -1 \end{cases}$$

$$-2|x+1|+2 = \begin{cases} -2x & , x \geq -1 \\ 2x+4 & , x < -1 \end{cases}$$

x	-3	-2	-1	0	1
y	-2	0	2	0	-2



Forth : Graph of Rational Function رسم الدالة الكسرية

نوجد أصفار المقام ، ثم نعوض عن قيم يمينها و يسارها ثم نصلها بحيث أن المنحنى يتقارب للخط المنقط (صفر المقام) لا يلمسه

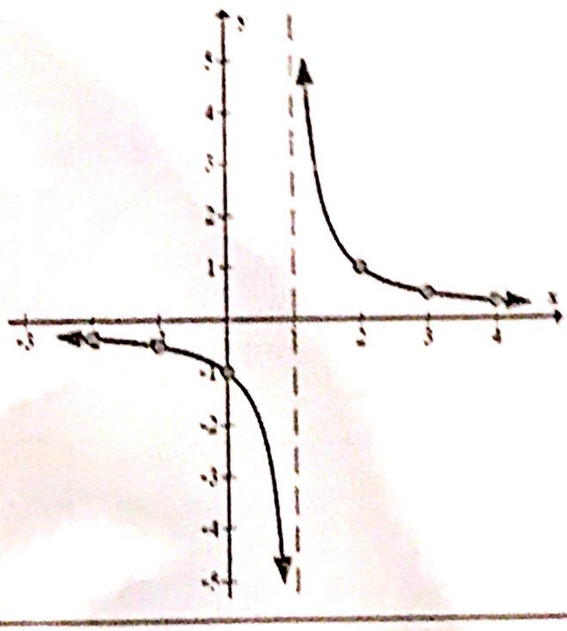
Related Problem (4)

1. $f(x) = \dfrac{1}{x-1}$

solution:

The zero of denominator : $x-1=0$, $x=1$

x	-2	-1	0	1	2	3	4
y	≈ -0.33	-0.5	-1	undefined	1	0.5	≈ 0.33



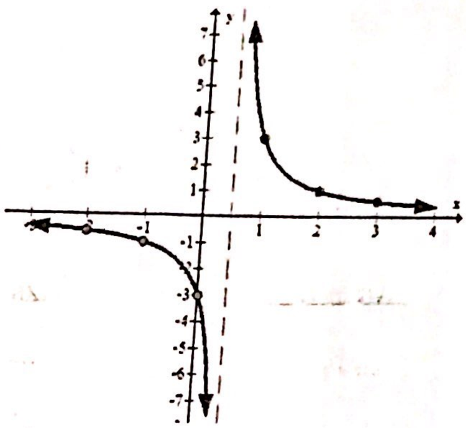
Exercise 14. $f(x)=\frac{3}{2x-1}$

solution:

The zeros of denominator : $2x-1=0$, $x=\frac{1}{2}$

Domain $D_f=(-\infty,\infty)-\left\{\frac{1}{2}\right\}$

x	-2	-1	0	$\frac{1}{2}$	1	2	3
y	-0.6	-1	-3	undefined	3	1	0.6



Fifth : Graph of Square Root Function رسم دالة الجذر التربيعي

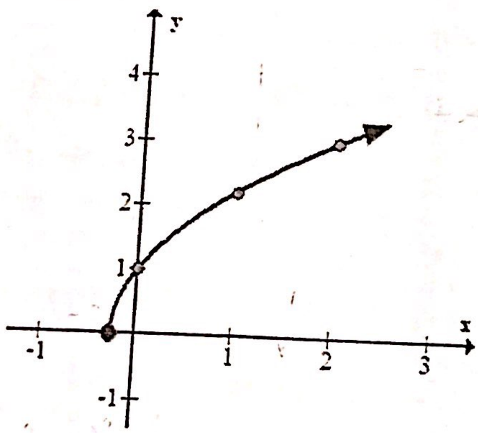
نوجد مجال الدالة و نكون جدول يحتوي نقاط في المجال و نعوض

Exercise 21. $f(x)=\sqrt{4x+1}$

solution:

$4x+1\geq 0$, $4x\geq -1$, $x\geq -\frac{1}{4}$

x	$-\frac{1}{4}$	0	1	2
y	0	1	≈ 2.2	3



Exercise 12. $y=\sqrt{x^2-1}$

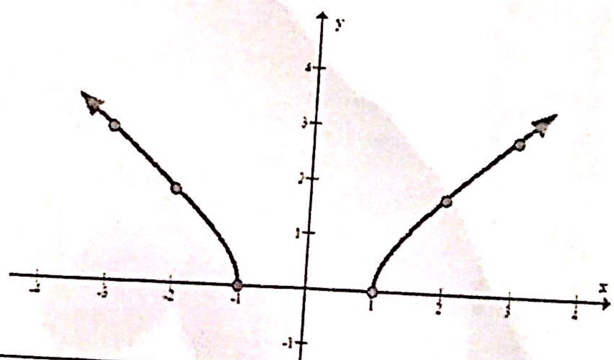
solution:

$x^2-1\geq 0$, $x^2\geq 1$

$\sqrt{x^2}\geq\sqrt{1}$, $|x|\geq 1$

$x\leq -1$ or $x\geq 1$

x	-3	-2	-1		1	2	3
y	2.8	1.7	0		0	1.7	2.8



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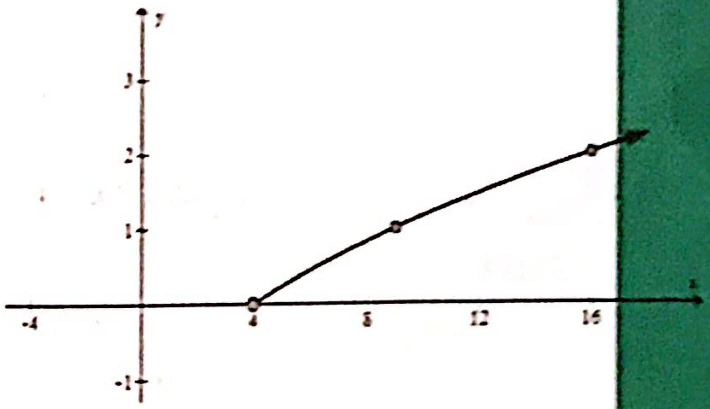
3. $f(x) = \sqrt{x} - 2 \quad , \quad x \geq 4$

solution:

x	4	9	16
y	0	1	2



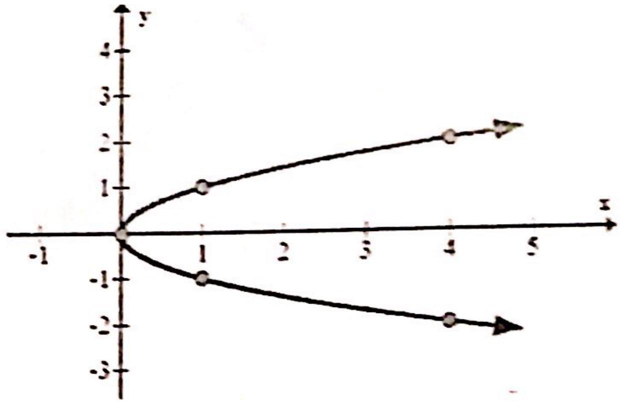
رسم بعض الدوال مطلوبة بالخطة

Exercises

Exercise 17. $x = y^2$

solution

x	0	1	4
y	0	± 1	± 2



Exercise 22. $f(x) = \begin{cases} x^2 & \text{for } 0 \leq x \leq 2 \\ 4 & \text{for } x > 2 \end{cases}$

تقسم الدالة جزئين و نرسم كل جزء

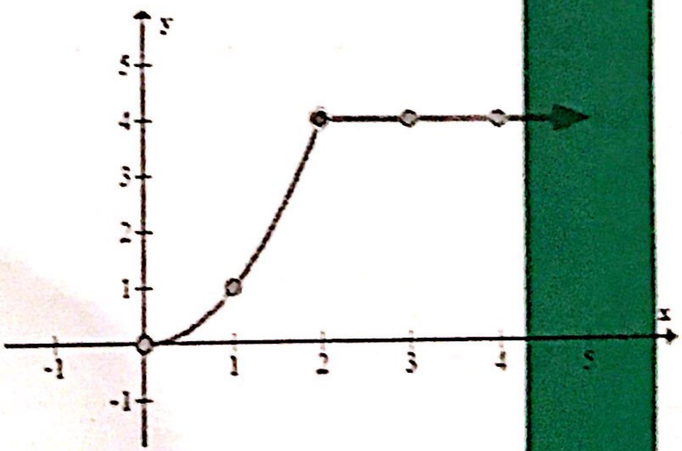
solution:

* Part 1: $f(x) = x^2 \quad \text{for} \quad 0 \leq x \leq 2$

x	0	1	2
y	0	1	4

* Part 2: $f(x) = 4 \quad , \quad x > 2$

x	2 (open point)	3	4
y	4	4	4



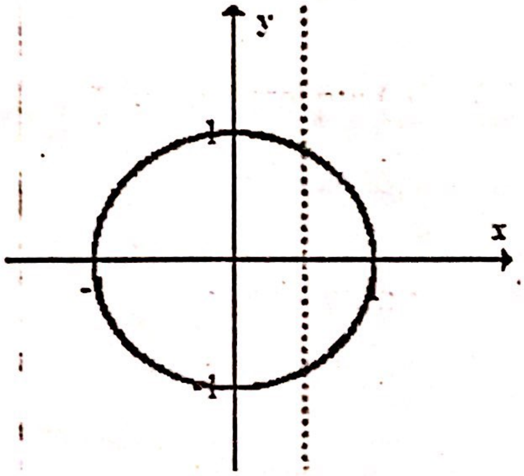
Exercises 29 – 41 , Sketch the graph of the equation . In each case determine whether the graph is that of a function

Exercise 29. $x^2 + y^2 = 1$

معادلة دائرة نصف قطرها (1) و مركزها نقطة الأصل (0,0)

solution:

$x^2 + y^2 = 1$ is a circle of raduis 1 centered at the origion $(0,0)$



The graph is not a function , because vertical lines cross the curve in more than point

ليست دالة، لان خطوط رأسية تقطع المنحنى في أكثر من نقطة

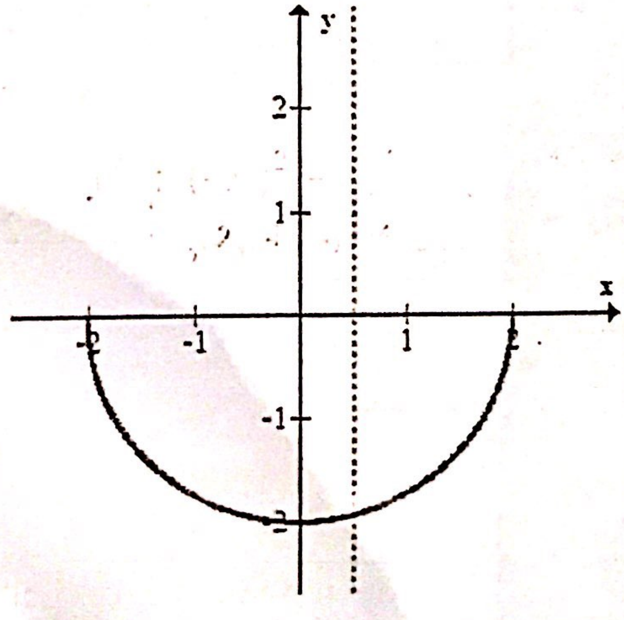
Exercise 32. $x^2 + y^2 = 4$ for $y \le 0$

معادلة دائرة نصف قطرها 2 و مركزها نقطة الأصل
لكن لاحظ $y < 0$ أي نصف الدائرة الأسفل

solution:

$x^2 + y^2 = 4$, $y \le 0$ is semi-circle

$$y = -\sqrt{4 - x^2}$$

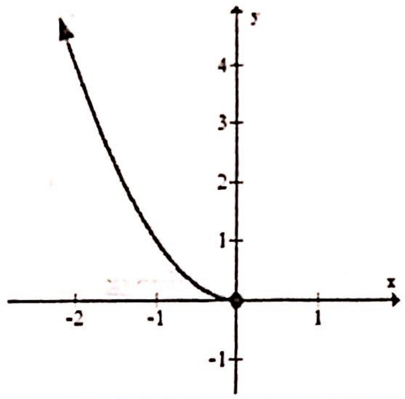


The graph is a function

Exercise 35. $y = x^2$ for $x \le 0$

solution:

x	-2	-1	0
y	4	1	0



The graph is a function

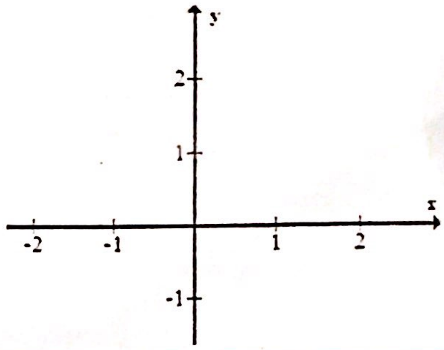
Exercise 39. $xy = 0$

solution:

$xy = 0 \Rightarrow x = 0$ or $y = 0$

The line $x = 0$ is y – axis

The line $y = 0$ is x – axis



The graph is not a function

41. $|x| + |y| = 1$

solution:

مجموع القيمتين المطلقة يساوي 1
نستنتج أن كل قيمة مطلقة أقل من 1

$|y| = 1 - |x|$

$y = \pm(1 - |x|)$

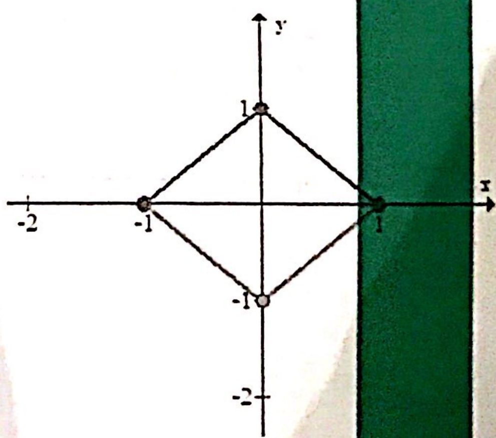
$y = 1 - |x|$ or $y = -(1 - |x|) = -1 + |x|$

Note: $|x| < 1$ and $|y| < 1$

* First : Sketch $y = 1 - |x|$ for $-1 \le x \le 1$

$$y = 1 - |x| = \begin{cases} 1 - x , & 0 \le x \le 1 \\ 1 + x , & -1 \le x < 0 \end{cases}$$

x	-1	0	1
y	0	1	0



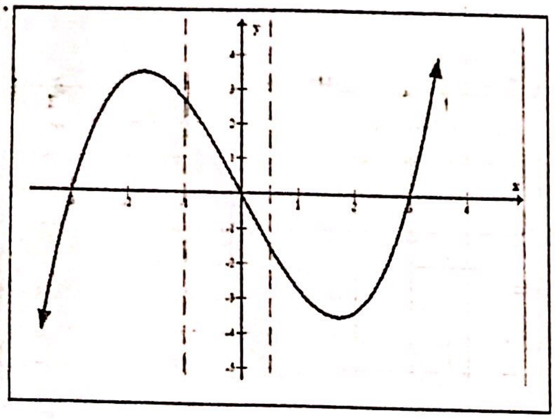
* Second : Sketch $y = -1 + |x|$ for $-1 \le x \le 1$

$$y = -1 + |x| = \begin{cases} -1 + x , & 0 \le x \le 1 \\ -1 - x , & -1 \le x < 0 \end{cases}$$

x	-1	0	1
y	0	-1	0

Exercises 24 – 28 , Determine which of the following curves represent a graph of a function . Explain your reason for any that do not define a function

25.



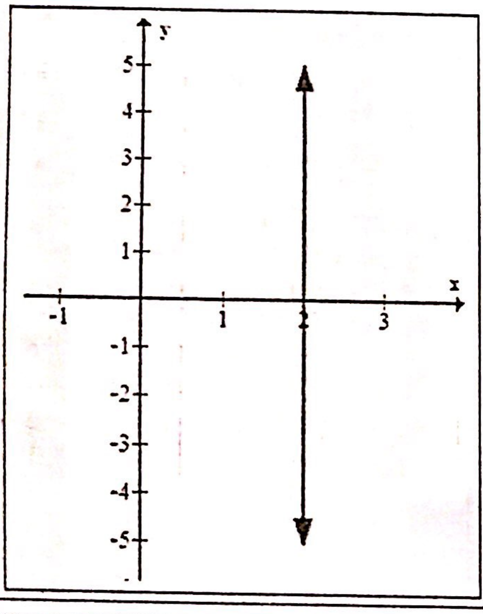
solution:

The curve defined a function ,

because every vertical line cross the curve in at most one point

دالة لان خطوط رأسية تقطع المنحنى على الأكثر في نقطة واحدة

26.



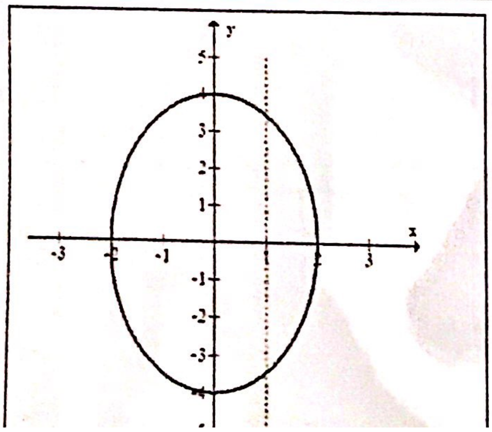
solution:

The curve is not a function.

Because vertical line $x = 2$ crosses the curve in more than one point

لأن الخط العمودي يقطع المنحنى في أكثر من نقطة

28.



Solution:

The curve is not a function,

Because vertical lines cross the curve in more than one point

Exercises (3.4) : AIDS TO GRAPHING مساعدات الرسم

تعاريف هامة

- x – intercept : نعوض عن $y = 0$ و نوجد قيم x
- x – intercept : نعوض عن $x = 0$ و نوجد قيم y
- Symmetric with respect to x – axis : نستبدل كل x بـ $-x$ نجد أن المعادلة الناتجة تكافئ المعادلة الأصلية
- Symmetric with respect to y – axis : نستبدل كل y بـ $-y$ نجد أن المعادلة الناتجة تكافئ المعادلة الأصلية
- Symmetric with respect to the origin : نستبدل كل من x و y بـ $-x$ و $-y$ نجد أن المعادلة الناتجة تكافئ المعادلة الأصلية

Exercises 1 – 16 , Determine all intercepts of the graph of the equation . Then decide whether the graph is symmetric x – axis , y – axis or the origin .

الأمثلة التالية :الكتاب المقرر شرح طريقتين لإثبات التماثل . تم الحل بالطريقة الأسهل

Exercise 1. $x = 3y^2 - 2$

solution:

* x – intercept ($y = 0$)

$x = -2$

* y – intercepts ($x = 0$)

$3y^2 - 2 = 0 \quad , \quad 3y^2 = 2 \quad , \quad y = \pm\sqrt{\frac{2}{3}}$

* Symmetric with x – axis

replace y by $(-y)$: $x = 3(-y)^2 - 2$

$x = 3y^2 - 2$

عند استبدال x بـ -x المعادلة الناتجة تكافئ المعادلة الأصلية

the graph is symmetric with x – axis

* Symmetric with y – axis

replace x by $-x$: $-x = 3y^2 - 2$. But is not true since $(-x) \neq x$

the graph is not symmetric with y – axis

عند استبدال y بـ -y المعادلة الناتجة لا تكافئ المعادلة الأصلية

* Symmetric with the origin

replace x and y by $-x$ and $-y$: $-x = 3(-y)^2 - 2$

$-x = 3y^2 - 2$. But is not true since $(-x) \neq x$

the graph is not symmetric with the origin

Exercise 4. $x^2 = y^{15} - y^9$

solution:

* x – intercept ($y = 0$)

$x = 0$

* y – intercepts ($x = 0$)

$y^{15} - y^9 = 0$, $y^9(y^6 - 1) = 0$,

$y^9 = 0$ or $y^6 - 1 = 0$

$y = 0$ or $y = 1$

* Symmetric with x – axis

replace y by $(-y)$: $x^2 = (-y)^{15} - (-y)^9$

$x^2 = -y^{15} + y^9$, But is not true since $(-y)^{15} \neq y^{15}$

the graph is not symmetric with x – axis

* Symmetric with y – axis

replace x by $-x$: $(-x)^2 = y^{15} - y^9$

$x^2 = y^{15} - y^9$

the graph is symmetric with y – axis

* Symmetric with the origin

replace x and y by $-x$ and $-y$: $(-x)^2 = (-y)^{15} - (-y)^9$

$x^2 = -y^{15} + y^9$. But is not true since $(-y)^{15} \neq y^{15}$

the graph is not symmetric with the origin

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Exercise 7. $x^2y^4 - 2x^4 = 1$

solution:

* x – intercept ($y = 0$)

$-2x^4 = 1$, $x^4 = -\frac{1}{2}$ has no real solution

No x – intercept

* y – intercepts ($x = 0$)

$0 = 1$, No y – intercept

* Symmetric with x – axis

replace y by $(-y)$: $x^2(-y)^4 - 2x^4 = 1$

$x^2y^4 - 2x^4 = 1$

the graph is symmetric with x – axis

* Symmetric with y – axis

replace x by $-x$: $(-x)^2y^4 - 2(-x)^4 = 1$

$x^2y^4 - 2x^4 = 1$

the graph is symmetric with y – axis

* Symmetric with the origin

replace x and y by $-x$ and $-y$: $(-x)^2(-y)^4 - 2(-x)^4 = 1$

$x^2y^4 - 2x^4 = 1$

the graph is symmetric with the origin

Exercise 10. $y = \frac{x}{1+x^2}$

solution:

* x – intercept ($y = 0$)

$$\frac{x}{1+x^2} = 0 \quad \Rightarrow \quad x = 0$$

* y – intercepts ($x = 0$)

$$y = \frac{(0)}{1+(0)^2} = 0$$

* Symmetric with x – axis

replace y by $(-y)$: $(-y) = \frac{x}{1+x^2}$, But is not true since $(-y) \neq y$

the graph is not symmetric with x – axis

* Symmetric with y – axis

replace x by $-x$: $y = \frac{(-x)}{1+(-x)^2}$

$$y = -\frac{x}{1+x^2}$$, But is not true since $(-x) \neq x$

the graph is not symmetric with y – axis

* Symmetric with the origin

replace x and y by $-x$ and $-y$: $(-y) = \frac{(-x)}{1+(-x)^2}$

$$-y = \frac{-x}{1+x^2}$$

$$y = \frac{x}{1+x^2}$$

the graph is symmetric with the origin

[redacted]11. $y=\sqrt{9-x^2}$

[redacted]on:

[redacted]rcept ($y=0$)

$\sqrt{9-x^2}=0$, $9-x^2=0$, $x^2=9$, $x=\pm 3$

[redacted]rcepts ($x=0$)

[redacted] $=\sqrt{9-0} \quad =3$

[redacted]tric with x – axis

[redacted]e y by $(-y)$: $(-y)=\sqrt{9-x^2}$, But is not true since $(-y)\neq y$

[redacted]ph is not symmetric with x – axis

[redacted]tric with y – axis

[redacted] by $-x$: $y=\sqrt{9-(-x)^2}$

$y=\sqrt{9-x^2}$

[redacted]ph is symmetric with y – axis

[redacted]etric with the origin

[redacted] and y by $-x$ and $-y$: $(-y)=\sqrt{9-(-x)^2}$

$-y=\sqrt{9-x^2}$

[redacted]ph is not symmetric with the origin

Exercise 12. $|x-3|=|y+5|$

solution:

* x – intercept ($y=0$)

$$|x-3|=5$$

$$x-3=5 \quad \text{or} \quad x-3=-5$$

$$x=8 \quad \text{or} \quad x=-2$$

* y – intercepts ($x=0$)

$$|y+5|=|-3|=3$$

$$y+5=3 \quad \text{or} \quad y+5=-3$$

$$y=-2 \quad \text{or} \quad y=-8$$

* Symmetric with x – axis

replace y by $(-y)$: $|x-3|=|-y+5|$, But is not true since $(-y)\neq y$

the graph is not symmetric with x – axis

* Symmetric with y – axis

replace x by $-x$: $|-x-3|=|y+5|$, But is not true since $(-x)\neq x$

the graph is not symmetric with y – axis

* Symmetric with the origin

replace x and y by $-x$ and $-y$: $|-x-3|=|-y+5|$

But is not true since $(-x)\neq x$, $(-y)\neq y$

the graph is not symmetric with the origin

Exercise 15. $y^2 = \frac{x^2+1}{x^2-1}$

solution:

* x – intercept ($y = 0$)

$$\frac{x^2+1}{x^2-1} = 0 \quad , \quad x^2+1 = 0 \quad \text{has no real solutions}$$

the graph has no x – intercept

* y – intercepts ($x = 0$)

$$y = \frac{0^2+1}{0^2-1} = -1$$

* Symmetric with x – axis

replace y by $(-y)$: $(-y)^2 = \frac{(-x)^2+1}{(-x)^2-1}$

$$y^2 = \frac{x^2+1}{x^2-1}$$

the graph is symmetric with x – axis

* Symmetric with y – axis

replace x by $-x$: $y^2 = \frac{(-x)^2+1}{(-x)^2-1}$

$$y^2 = \frac{x^2+1}{x^2-1}$$

the graph is symmetric with y – axis

* Symmetric with the origin

replace x and y by $-x$ and $-y$: $(-y)^2 = \frac{(-x)^2+1}{(-x)^2-1}$

$$y^2 = \frac{x^2+1}{x^2-1}$$

the graph is symmetric with the origin

Exercise 16. $y = x^2\sqrt{9-x^4}$

solution:

* x – intercept ($y = 0$)

$$x^2\sqrt{9-x^4} = 0 \quad ,$$

$$x^2 = 0 \quad \text{or} \quad 9-x^4 = 0$$

$$x = 0 \quad \text{or} \quad (3-x^2)(3+x^2) = 0 \qquad x = \pm 3$$

$$x = 0 \quad \text{or} \quad (3-x^2) = 0 \ , \quad (3+x^2) = 0 \ \text{(no solutions)}$$

$$x = \pm\sqrt{3}$$

x – intercepts are $x = 0$, $x = -\sqrt{3}$, $x = \sqrt{3}$

* y – intercepts ($x = 0$)

$$y = 0^2\sqrt{9-0^4} = 0$$

* Symmetric with x – axis

replace y by $(-y)$: $\quad (-y) = x^2\sqrt{9-x^4}$, But is not true since $(-y) \neq y$

the graph is not symmetric with x – axis

* Symmetric with y – axis

replace x by $-x$: $\quad y = (-x)^2\sqrt{9-(-x)^4}$

$$y = x^2\sqrt{9-x^4}$$

the graph is symmetric with y – axis

* Symmetric with the origin

replace x and y by $-x$ and $-y$: $(-y) = (-x)^2\sqrt{9-(-x)^4}$

$$-y = x^2\sqrt{9-x^4} \ ,$$

But is not true since $(-y) \neq y$

the graph is not symmetric with the origin

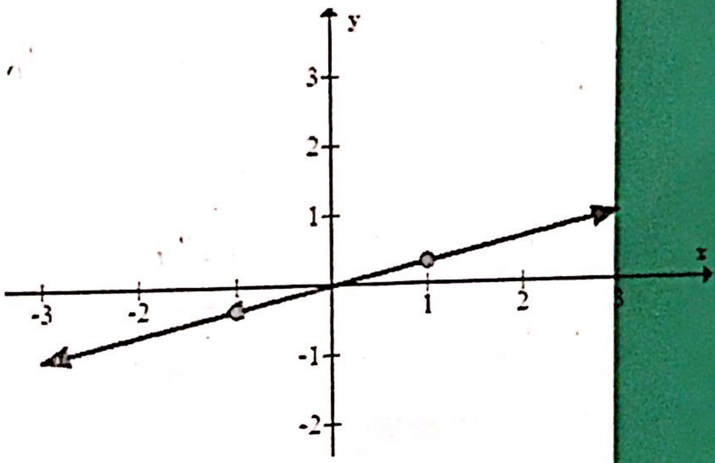
Exercises 17 – 26 , Sketch the graph . List the intercepts and describe the symmetry (if any) of the graph

17. $y = \frac{1}{3}x$

solution:

$y = \frac{1}{3}x$ is a linear equation

x	-1	1
y	$-\frac{1}{3}$	$\frac{1}{3}$



* x – intercept : $x = 0$

* y – intercept : $y = 0$

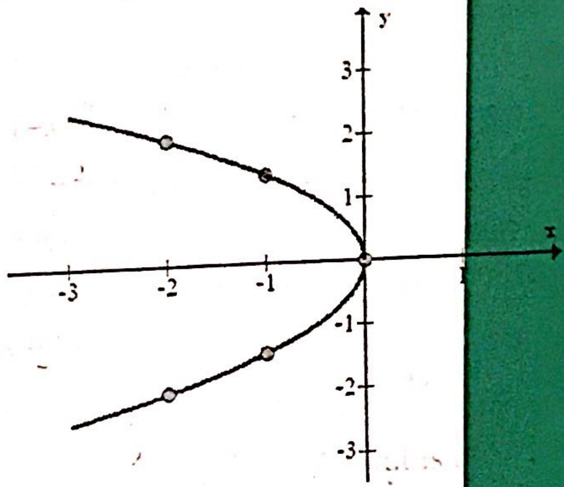
* The graph is symmetric with respect to the origin

Exercise 18. $2x = -y^2$

solution:

$x = -\frac{1}{2}y^2$

x	-2	-1	0
y	± 2	± 1.4	0



* x – intercept : $x = 0$

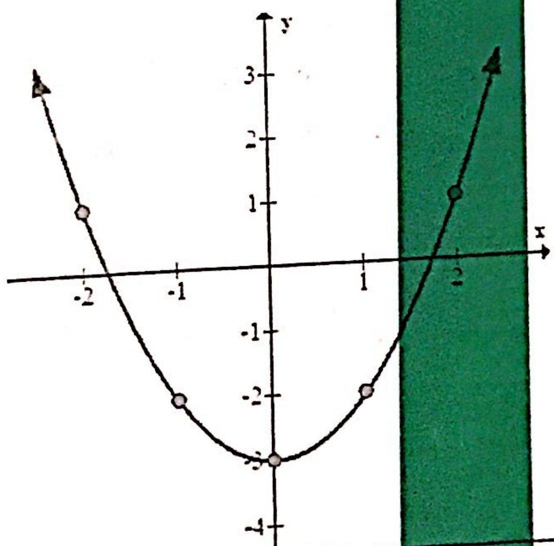
* y – intercept : $y = 0$

* The graph is symmetric with respect to x – axis

Exercise 19. $y = x^2 - 3$

solution:

x	-2	-1	0	1	2
y	1	-2	-3	-2	1



* x – intercept ($y = 0$) : $x = \pm\sqrt{3}$

* y – intercept ($x = 0$) : $y = -3$

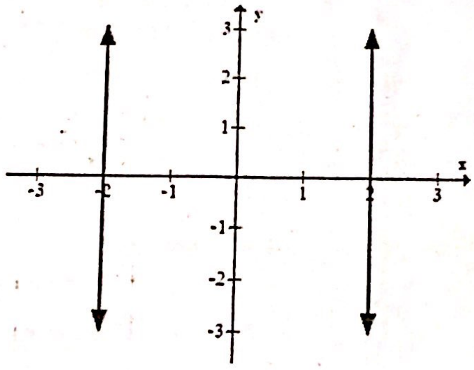
* The graph is symmetric with respect to y – axis

Exercise 20. $|x|=2$

solution:

Lines: $x=2$ or $x=-2$

* x – intercept : $x=-2$, $x=2$
* y – intercept : has no y – intercept
* The graph is symmetric with respect with x – axis , y – axis and the origin

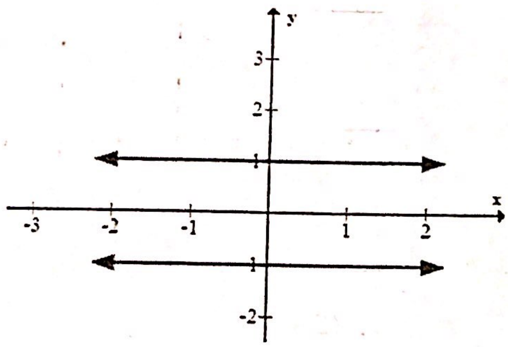


Exercise 21. $|y|=1$

solution:

Lines : $y=1$, $y=-1$

* x – intercept : No x – intercept
* y – intercept : $y=-1$, $y=1$
* The graph is symmetric with respect to x – axis , y – axis and the origin



Exercise 24. $y=\sqrt{25-x^2}$

نوجد مجال الدالة و نعوض بنقاط بالمجال و نرسمها

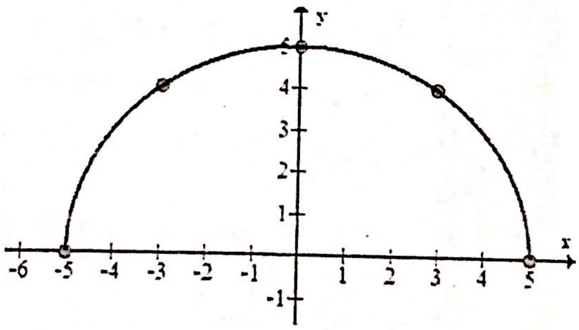
solution:

$25-x^2\geq 0$, $-x^2\geq -25$, $x^2\leq 25$, $|x|\leq 5$

Define for $-5\leq x\leq 5$

x	-5	-3	0	3	5
y	0	4	5	4	0

* x – intercept : $x=-5$, $x=5$
* y – intercept : $y=5$
* The graph is symmetric with respect to y – axis



Even and Odd Functions الدوال الزوجية و الفردية

- Even Function : If $f(-x) = f(x)$
- Odd Function : If $f(-x) = -f(x)$

تم إضافة هذا المثال من Related Problem لتكون المذكرة شاملة أفكار المنهج

Related Problem(7)

Ex : Determine algebraically whether the following functions are even , odd, or neither

1. $f(x) = x^2 + 3$ 2. $f(x) = \dfrac{2x - x^5}{x^4 + 1}$ 3. $h(x) = x^3 + x^2$

solution:

1.

$$f(-x) = (-x)^2 + 3$$
$$= x^2 + 3$$
$$= f(x)$$

f is even

2.

$$f(-x) = \frac{2(-x) - (-x)^5}{(-x)^4 + 1}$$
$$= \frac{-2x + x^5}{x^4 + 1}$$
$$= \frac{-(2x - x^5)}{x^4 + 1}$$
$$= -f(x)$$

f is odd

3.

$$h(-x) = (-x)^3 + (-x)^2$$
$$= -x^3 + x^2$$
$$= -(x^3 - x^2)$$
$$\neq -h(x)$$

h is neither even nor odd

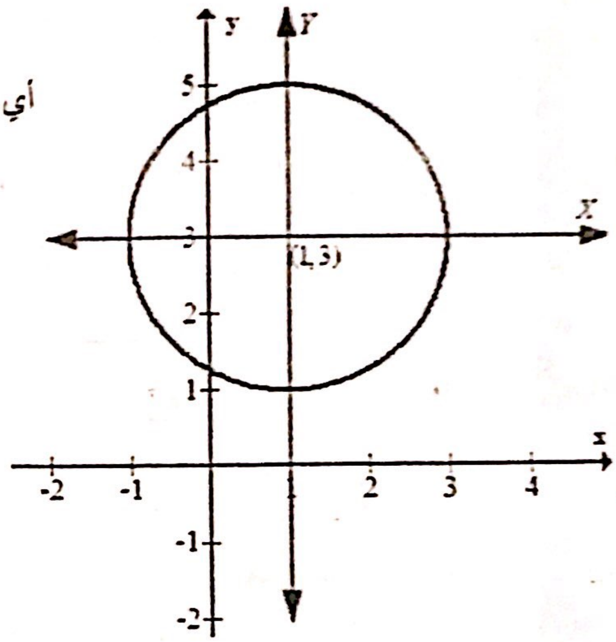
Exercises 27 – 38 , Sketch the graph of the given equation with the help of a suitable translation . Show both the x and y axis , and the X and Y axis

Exercise 27. $(x-1)^2+(y-3)^2=4$

solution:

* Let $X=x-1$ and $Y=y-3$
* The origin of XY coordinate $(h,k)=(1,3)$
* The equation in XY coordinate : $X^2+Y^2=4$ is a circle of raduis $=2$

أي نرسم دائرة نصف قطرها 2 ولكن مركزها (1,3)



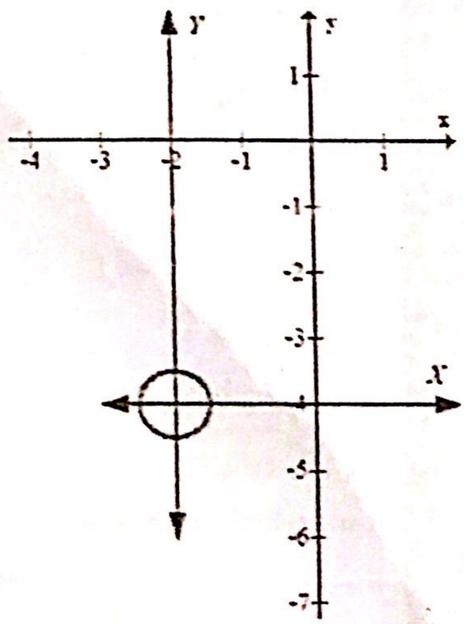
Exercise 28. $(x+2)^2+(y+4)^2=\frac{1}{4}$

solution:

* Let $X=x+2$ and $Y=y+4$
* The origin of XY coordinate $(h,k)=(-2,-4)$
* The equation in XY coordinate :

$X^2+Y^2=\frac{1}{4}$ is a circle of raduis $=\frac{1}{2}$

أي نرسم دائرة نصف قطرها 1/2 ولكن مركزها (-2,-4)



Exercise 30. $x^2 + y^2 + 4y = -1$

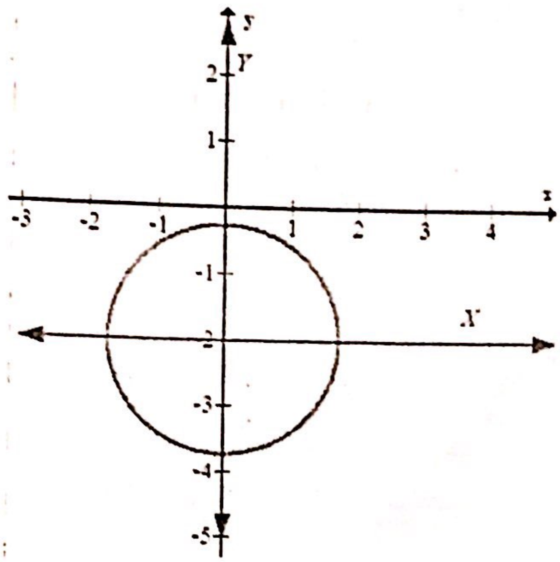
solution:

$$x^2 + y^2 + 4y + (2)^2 = -1 + (2)^2$$

اكمال مربع لحدود y

$$x^2 + (y + 2)^2 = 3$$

* Let $X = x$ and $Y = y + 2$
* The origin of XY coordinate $(h, k) = (0, -2)$
* The equation in XY coordinate : $X^2 + Y^2 = 3$ is a circle of raduis $= \sqrt{3}$



Exrcise 33. $x^2 + y^2 + 4x - 6y + 13 = 0$

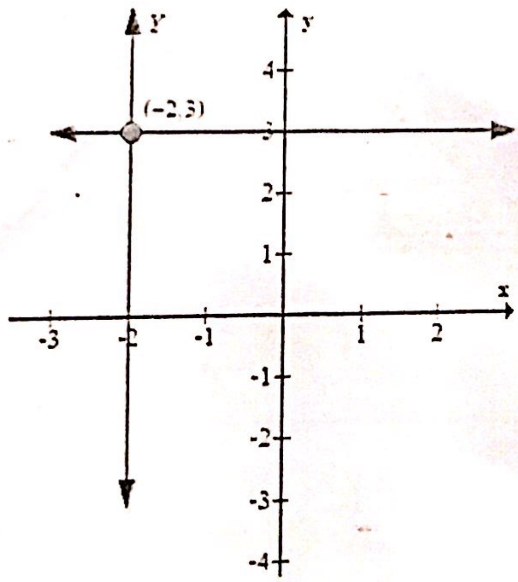
solution:

$$x^2 + 4x + (2)^2 + y^2 - 6y + (3)^2 = -13 + (2)^2 + (3)^2$$

اكمال مربع لحدود x و y

$$(x + 2)^2 + (y - 3)^2 = 0$$

* Let $X = x + 2$ and $Y = y - 3$
* The origin of XY coordinate $(h, k) = (-2, 3)$
* The equation in XY coordinate system : $X^2 + Y^2 = 0$ is a point



Exercise 34. $x^2-6x+y^2-y=-9$

solution:

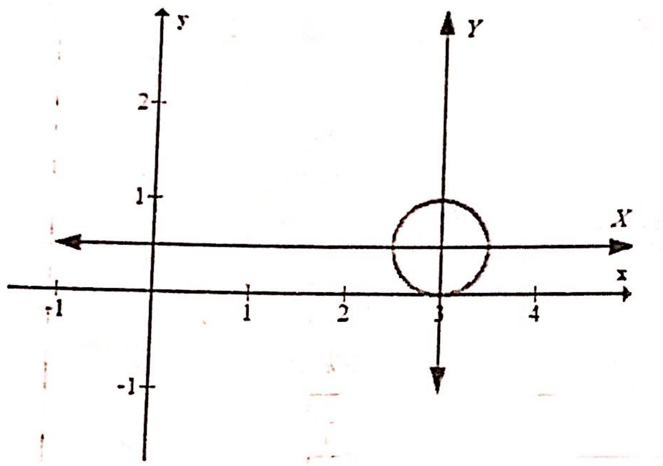
$$x^2-6x+(3)^2+y^2-y+\left(\frac{1}{2}\right)^2=-9+(3)^2+\left(\frac{1}{2}\right)^2$$

$$(x-3)^2+(y-\frac{1}{2})^2=\frac{1}{4}$$

* Let $X=x-3$ and $Y=y-\frac{1}{2}$

* The origin of XY coordinate $(h,k)=(3,\frac{1}{2})$

* The equation in XY coordinate system : $X^2+Y^2=\frac{1}{4}$ is a circle, raduis is $\frac{1}{2}$



EXercise 36. $y=|x-4|$

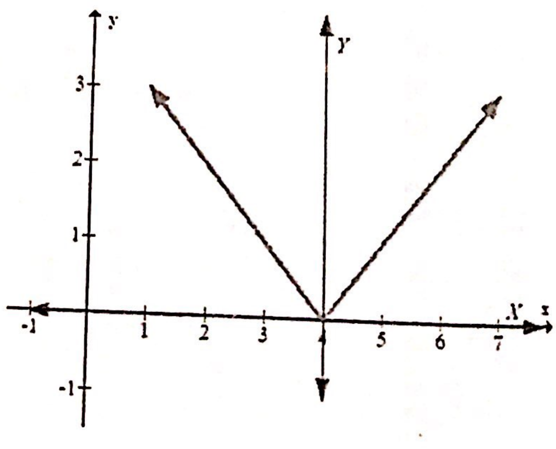
solution:

* Let $X=x-4$ and $Y=y$

* The origin of XY coordinate $(h,k)=(0,4)$

* The equation in XY coordinate : $Y=|X|$

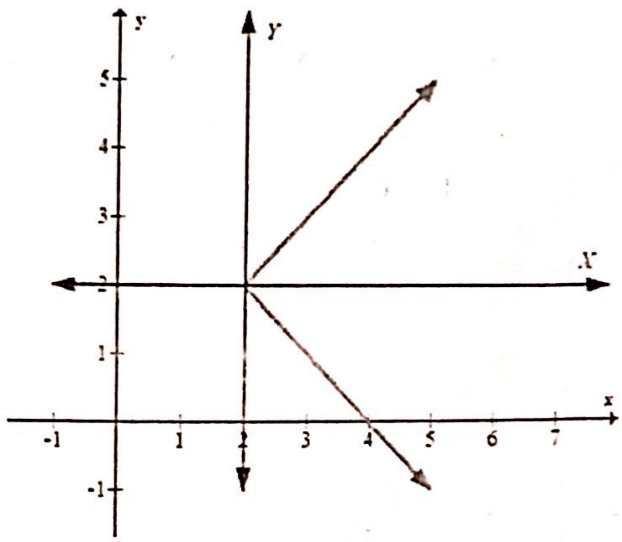
نرسم دالة القيمة المطلقة ولكن نقطة الرأس (0,4)



Exercise 67. $x-2=|y-2|$

solution:

* Let $X=x-2$ and $Y=y-2$
* The origin of XY coordinate $(h,k)=(2,2)$
* The equation in XY coordinate : $X=|Y|$



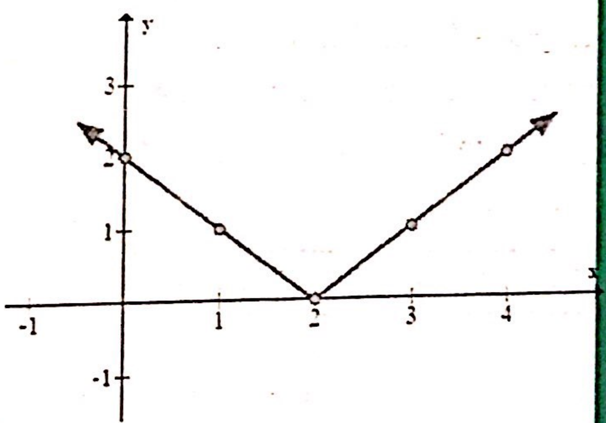
Exercise 49. Let $f(x)=|x|$ and $g(x)=f(x-2)$. Sketch the graph of g

solution:

$g(x)=f(x-2)=|x-2|$

$$=\begin{cases}(x-2) & , x\geq 2\\ -(x-2) & , x<2\end{cases}$$

x	0	1	2	3	4
y	2	1	0	1	2



EXERCISES 3.5 : COMBINING FUNCTIONS

Exercises 1 – 10 , Let $f(x) = x^2 + 4x - 2$ and $g(x) = 2 - x^2$. Find the specified values

1. $(f+g)(-1)$

   solution:

   * $(f+g)(-1) = f(-1) + g(-1)$

   $$= \left((-1)^2 + 4(-1) - 2\right) + \left(2 - (-1)^2\right)$$

   $$= 3 + 1 = 4$$

2. $(f-g)(2)$

   solution:

   * $(f-g)(2) = f(2) - g(2)$

   $$= \left((2)^2 + 4(2) - 2\right) + \left(2 - (2)^2\right)$$

   $$= 10 + -2 = 8$$

4. $(f \cdot g)(0)$

   solution:

   * $(f \cdot g)(0) = f(0) \cdot g(0)$

   $$= \left((0)^2 + 4(0) - 2\right) \cdot \left(2 - (0)^2\right)$$

   $$= (-2) \cdot (2) = -4$$

5. $\left(\frac{f}{g}\right)(1)$

   solution:

   * $\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)}$

   $$= \frac{(1)^2 + 4(1) - 2}{2 - (1)^2}$$

   $$= \frac{3}{1} = 3$$

6. $(f \circ g)(3)$

solution:

$$* \ (f \circ g)(3) = f\left(g(3)\right)$$
$$= f\left(2 - 3^2\right)$$
$$= f(-7)$$
$$= (-7)^2 + 4(-7) - 2 = 19$$

9. $(g \circ g)(2)$

solution:

$$* \ (g \circ g)(2) = g\left(g(2)\right)$$
$$= g\left(2 - 2^2\right)$$
$$= g(-2)$$
$$= 2 - (-2)^2 = -2$$

Exercises 11 – 15 , Let $f(x) = \dfrac{x-1}{x^2+2}$ and $g(x) = (x)^{1/4}$. Find the specified values.

12. $(g / f)(x)$

solution:

$$* \ (g/f)(x) = \frac{g(x)}{f(x)}$$
$$= \frac{(x)^{1/4}}{\frac{x-1}{x^2+2}} = (x)^{1/4} \div \frac{x-1}{x^2+2}$$
$$= (x)^{1/4} \cdot \frac{(x^2+2)}{x-1}$$

14. $(f \circ g)(x)$

solution

$$* \ (f \circ g)(x) = f\left(g(x)\right)$$
$$= f\left(x^{1/4}\right)$$
$$= \frac{x^{1/4} - 1}{\left(x^{1/4}\right)^2 + 2} = \frac{x^{1/4} - 1}{x^{1/2} + 2}$$

15. $(g \circ g)(x)$

solution:

* $(g \circ g)(x) = g(g(x))$

$$= g\left(x^{1/4}\right)$$

$$= \left(x^{1/4}\right)^{1/4}$$

$$= x^{1/16}$$

Theorem:

Let f and g be functions

1. $D_{f+g} = D_f \cap D_g$
2. $D_{f-g} = D_f \cap D_g$
3. $D_{f \cdot g} = D_f \cap D_g$
4. $D_{f/g} = D_f \cap D_g - \{g(x) = 0\}$

Exercises $16-24$, Find the domain and rules of $f+g$, $f \cdot g$ and $\frac{f}{g}$

20. $f(t) = t^{3/4}$; $g(t) = t^2 + 3$

solution:

نلاحظ $t^{\frac{3}{4}}$ تعتبر $\sqrt[4]{t^3}$ الجذر الرابع مجاله نفس الجذر التربيعي

Domain of f : $t \geq 0$, $[0, \infty)$

Domain of g : $(-\infty, \infty)$

The zeros of $g(t)$: $t^2 + 3 = 0$ has no real solution

* $(f+g)(t) = f(t) + g(t)$

$$= t^{3/4} + t^2 + 3$$

$D_{f+g} = D_f \cap D_g$

$$= [0, \infty) \cap (-\infty, \infty) = [0, \infty)$$

* $(f \cdot g)(t) = f(t) \cdot g(t)$

$$= t^{3/4} \cdot (t^2 + 3)$$

$D_{f \cdot g} = D_f \cap D_g$

$$= [0, \infty) \cap (-\infty, \infty) = [0, \infty)$$

* $\left(\frac{f}{g}\right)(t) = \frac{f(t)}{g(t)} = \frac{t^{3/4}}{t^2 + 3}$

$D_{f/g} = D_f \cap D_g - \{g(x) = 0\}$

$$= [0, \infty)$$

21. $f(x)=\sqrt{1-x^2}\ ;\ g(t)=\sqrt{2+x-x^2}$

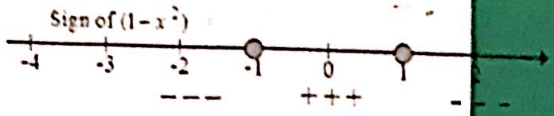
مجال الجذر التربيعي داخله دالة تربيعية :
نضع أصفار الدالة على خط الأعداد و نبحث الاشارة
نبحث عن الفترات التي لها الاشارة موجب

solution:

Domain of f : $1-x^2\ge 0$

$(1-x)(1+x)\ge 0$

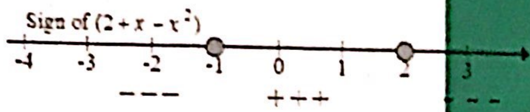
$D_f=[-1,1]$



Domain of g : $2+x-x^2\ge 0$

$-(x^2-x-2)\ge 0\ ,\ -(x-2)(x+1)\ge 0$

$D_g=[-1,2]$



The zeros of $g(x)$: $2+x-x^2=0\ \Rightarrow\ x=-1\ ,\ x=2$

* $(f+g)(x)=f(x)+g(x)$

$=\sqrt{1-x^2}+\sqrt{2+x-x^2}$

$D_{f+g}=D_f\cap D_g$

$=[-1,1]\cap[-1,2]$

$=[-1,1]$

* $(f\cdot g)(x)=f(x)\cdot g(x)$

$=\sqrt{1-x^2}\cdot\sqrt{2+x-x^2}$

$=\sqrt{2+x-x^2-2x^2-x^3+x^4}=\sqrt{2+x-3x^2-x^3+x^4}$

$D_{f\cdot g}=D_f\cap D_g$

$=[-1,1]\cap[-1,2]$

$=[-1,1]$

* $\left(\dfrac{f}{g}\right)(x)=\dfrac{f(x)}{g(x)}$

$=\dfrac{\sqrt{1-x^2}}{\sqrt{2+x-x^2}}$

$=\sqrt{\dfrac{(1-x)(1+x)}{-(x-2)(x+1)}}=\sqrt{\dfrac{1-x}{2-x}}$

$D_{f/g}=D_f\cap D_g-\{g(x)=0\}$

$=[-1,1]-\{-1,2\}=(-1,1]$

24. $f(x)=\dfrac{x-2}{x+6}$; $g(t)=\dfrac{1}{\sqrt{x}}$

solution:

Domain of f : $x+6\neq 0$, $x\neq -6$

$$D_f=(-\infty,\infty)-\{-6\}$$

Domain of g : $x>0$

$$D_g=(0,\infty)$$

The zeros of $g(x)$: $\dfrac{1}{\sqrt{x}}=0$ has no zeros

* $(f+g)(x)=f(x)+g(x)$

$$=\frac{x-2}{x+6}+\frac{1}{\sqrt{x}}$$

$$D_{f+g}=D_f\cap D_g$$
$$=(-\infty,\infty)-\{-6\}\cap(0,\infty)$$
$$=(0,\infty)$$

* $(f\cdot g)(x)=f(x)\cdot g(x)$

$$=\frac{x-2}{x+6}\cdot\frac{1}{\sqrt{x}}$$

$$D_{f\cdot g}=D_f\cap D_g$$
$$=(-\infty,\infty)-\{-6\}\cap(0,\infty)$$
$$=(0,\infty)$$

* $\left(\dfrac{f}{g}\right)(x)=\dfrac{f(x)}{g(x)}$

$$=\frac{\frac{x-2}{x+6}}{\frac{1}{\sqrt{x}}}=\frac{(x-2)\sqrt{x}}{x+6}$$

$$D_{f/g}=D_f\cap D_g-\{g(x)=0\}$$
$$=(-\infty,\infty)-\{-6\}\cap(0,\infty)$$
$$=(0,\infty)$$

Domain of Composition of Functions

1. Domain of $f \circ g$: is Domain of $g \cap \{x : g(x) \in f(x)\}$

خطوات ايجاد مجال $f \circ g$

1- نوجد مجال g
2- نوجد عملية التحصيل $f \circ g$ ثم نوجد مجال الدالة الناتجة
3- نوجد تقاطع بين الفترات الناتجة في الخطوة الأولى والثانية

2. Domain of $g \circ f$: is Domain of $f \cap \{x : f(x) \in g(x)\}$

خطوات ايجاد مجال $g \circ f$

1- نوجد مجال f
2- نوجد عملية التحصيل $g \circ f$ ثم نوجد مجال الدالة الناتجة
3- نوجد تقاطع بين الفترات الناتجة في الخطوة الأولى والثانية

Exercises 25–33 , find the domains and rules of $g \circ f$, and $f \circ g$

25. $f(x) = 1 - x$; $g(x) = 2x + 5$

solution:

$D_f = \mathbb{R}$

$D_g = \mathbb{R}$

1. $(g \circ f)(x) = g(f(x))$

$= g(1 - x)$

$= 2(1 - x) + 5$

$= 2 - 2x + 5 = -2x + 7$ (Domain of $-2x + 7$ is $\mathbb{R}$)

Domain of $(g \circ f)$ = Domain of $f \cap$ Domain of $-2x + 7$

$= (-\infty, \infty) \cap (-\infty, \infty) = (-\infty, \infty)$

2. $(f \circ g)(x) = f(g(x))$

$= f(2x + 5)$

$= 1 - (2x + 5)$

$= 1 - 2x - 5 = -2x - 4$ (Domain of $-2x - 4$ is $\mathbb{R}$)

Domain of $(f \circ g)$ = Domain of $g \cap$ Domain of $-2x - 7$

$= (-\infty, \infty) \cap (-\infty, \infty) = (-\infty, \infty)$

27. $f(x)=x^2$; $g(x)=\sqrt{x}$

solution:

Domain of f : $\mathbb{R}$

Domain of g : $x \geq 0$, $[0,\infty)$

1. $(g \circ f)(x)=g(f(x))$

$$=g\left(x^2\right)=\sqrt{x^2}=|x| \qquad \left(\text{ Domain of } |x| \text{ is } \mathbb{R}\right)$$

Domain of $g \circ f$ = Domain of f $\cap$ Domain of $|x|$

$$=\mathbb{R}\cap\mathbb{R}=\mathbb{R}$$

2. $(f \circ g)(x)=f(g(x))$

$$=f\left(\sqrt{x}\right)=(\sqrt{x})^2=x \qquad \left(\text{ Domain of } x \text{ is } \mathbb{R}\right)$$

Domain of $f \circ g$ = Domain of g $\cap$ Domain of x

$$=[0,\infty)\cap\mathbb{R}=[0,\infty)$$

30. $f(x)=\frac{1}{x}$; $g(x)=x^2-3x-10$

solution:

Domain of f : $x \neq 0$, $(-\infty,0)\cup(0,\infty)$

Domain of g : $\mathbb{R}$

1. $(g \circ f)(x)=g(f(x))$

$$=g\left(\frac{1}{x}\right)$$

$$=\left(\frac{1}{x}\right)^2-3\left(\frac{1}{x}\right)-10$$

$$=\frac{1}{x^2}-\frac{3}{x}-10 \qquad \left(\text{ Domain of } \frac{1}{x^2}-\frac{3}{x}-10 \text{ is } x \neq 0\right)$$

Domain of $g \circ f$ = Domain of f $\cap$ Domain of $\frac{1}{x^2}-\frac{3}{x}-10$

$$=(-\infty,0)\cup(0,\infty)$$

2. $(f \circ g)(x)=f(g(x))$

$$=f\left(x^2-3x-10\right)$$

$$=\frac{1}{x^2-3x-10}$$

$$=\frac{1}{(x-5)(x+2)} \qquad \left(\text{ Domain of } \frac{1}{(x-5)(x+2)} \text{ is } x \neq 5\text{ , } x \neq -2\right)$$

Domain of $f \circ g$ = Domain of g $\cap$ Domain of $\frac{1}{(x-5)(x+2)}$

$$=(-\infty,-2)\cup(-2,5)\cup(5,\infty)$$

32. $f(x)=\frac{1}{x-1}$; $g(x)=\frac{1}{x+1}$

solution:

Domain of $f: x \neq 1$, $(-\infty,\infty)-\{1\}$

Domain of $g: x \neq -1$, $(-\infty,\infty)-\{-1\}$

1. $(g \circ f)(x)=g(f(x))$

$$=g\left(\frac{1}{x-1}\right)$$

$$=\frac{1}{\frac{1}{x-1}+1} \cdot \frac{x-1}{x-1}$$

$$=\frac{x-1}{1+x-1}$$

$$=\frac{x-1}{x} \quad \left(\text{Domain of } \frac{x-1}{x} \text{ is } x \neq 0\right)$$

Domain of $g \circ f$ = Domain of f $\cap$ Domain of $\frac{x-1}{x}$

$$=\mathbb{R}-\{0,1\}$$

$$=(-\infty,0)\cup(0,1)\cup(1,\infty)$$

2. $(f \circ g)(x)=f(g(x))$

$$=f\left(\frac{1}{x+1}\right)$$

$$=\frac{1}{\frac{1}{x+1}+1} \cdot \frac{x+1}{x+1}$$

$$=\frac{x+1}{1+x+1}$$

$$=\frac{x+1}{x+2} \quad \left(\text{Domain of } \frac{x+1}{x+2} \text{ is } x \neq -2\right)$$

Domain of $f \circ g$ = Domain of g $\cap$ Domain of $\frac{x+1}{x+2}$

$$=\mathbb{R}-\{-2,-1\}$$

$$=(-\infty,-2)\cup(-2,-1)\cup(-1,\infty)$$

33. $f(x)=\sqrt{x^2+3}$; $g(x)=\sqrt{x^2-4}$

solution:

Domain of f : $x^2+3\ge 0$, $x^2\ge -3 \Rightarrow (-\infty,\infty)$

Domain of g : $x^2-4\ge 0$, $x^2\ge 4$, $\sqrt{x^2}\ge\sqrt{4}$, $|x|\ge 2$

$x\le -2$ or $x\ge 2 \Rightarrow (-\infty,-2)\cup(2,\infty)$

1. $(g\circ f)(x)=g(f(x))$

$$=g\left(\sqrt{x^2+3}\right)$$

$$=\sqrt{\left(\sqrt{x^2+3}\right)^2-4}$$

$$=\sqrt{x^2+3-4}$$

$$=\sqrt{x^2-1}$$

$\left(\text{Domain of } \sqrt{x^2-1} \text{ is } x^2-1\ge 0,\ |x|\ge 1,\ x\le -1,\ x\ge 1 \Rightarrow (-\infty,-1)\cup(1,\infty)\right)$

Domain of $g\circ f$ = Domain of f $\cap$ Domain of $\sqrt{x^2-1}$

$$=(-\infty,\infty)\ \cap\ ((-\infty,-1)\cup(1,\infty))$$

$$=(-\infty,-1)\cup(1,\infty)$$

2. $(f\circ g)(x)=f(g(x))$

$$=f\left(\sqrt{x^2-4}\right)$$

$$=\sqrt{\left(\sqrt{x^2-4}\right)^2+3}$$

$$=\sqrt{x^2-4+3}$$

$$=\sqrt{x^2-1}$$

$\left(\text{Domain of } \sqrt{x^2-1} \text{ is } x^2-1\ge 0,\ |x|\ge 1,\ x\le -1,\ x\ge 1 \Rightarrow (-\infty,-1)\cup(1,\infty)\right)$

Domain of $f\circ g$ = Domain of g $\cap$ Domain of $\sqrt{x^2-1}$

$$=(-\infty,-2)\cup(2,\infty)\ \cap\ (-\infty,-1)\cup(1,\infty)$$

$$=(-\infty,-2)\cup(2,\infty)$$

Exercises 34 – 41 , Write F as the composite $g \circ f$ of two functions f and g (neither of which equal to F)

35. $F(x) = \sqrt{x+2}$

solution:

$f(x) = x + 2$, $g(x) = \sqrt{x}$

38. $F(x) = |2x + 9|$

solution

$f(x) = 2x + 9$, $g(x) = |x|$

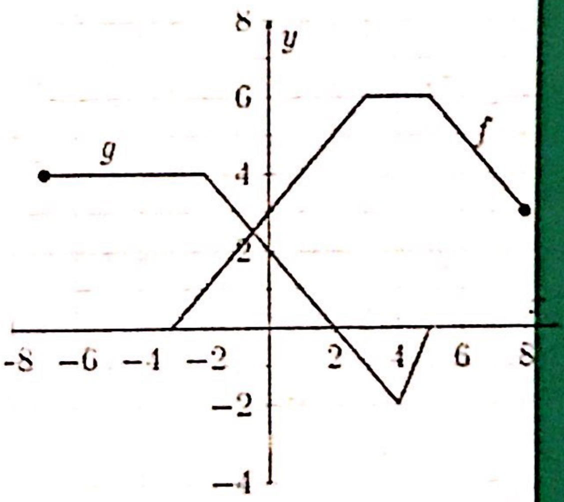
40. $F(x) = \dfrac{2}{x-3}$

solution:

$f(x) = x - 3$, $g(x) = \dfrac{2}{x}$

Related Problem(7)

Ex : Find te values of each of the following by using the graph.



1. $(f + g)(2) = f(2) + g(2)$
$= 5 + 0 = 5$

2. $(f - g)(4) = f(4) - g(4)$
$= 6 - (-2) = 8$

3. $(f \cdot g)(-2) = f(-2) \cdot g(-2)$
$= (1) \cdot (4) = 4$

4. $\left(\dfrac{g}{f}\right)(5) = \dfrac{g(5)}{f(5)}$
$= \dfrac{0}{6} = 0$

5. $(g \circ f)(-3) = g(f(-3))$
$= g(0) = 2$

6. $(f \circ f)(2) = f(f(2))$
$= f(5) = 6$

7. $(g \circ g)(1) = g(g(1))$
$= g(1) = 1$

Example and related problem (9)

Ex : Given that f is even function and g is odd function. Determine whether h is even , odd or neither

1. $h(x)=2f(x)+xg(x)$
2. $h(x)=xf(x)+g(x)$
3. $h(x)=f(x)\cdot g(x)$
4. $h(x)=(x^2+1)f(x)+g(x)$

solution

f is even $\Rightarrow f(-x)=f(x)$

g is odd $\Rightarrow g(-x)=-g(x)$

1. $h(-x)=2f(-x)+(-x)g(-x)$
$=2f(x)+(-x)(-g(x))$
$=2f(x)+xg(x)$
$=h(x)$, $h(x)$ is even

2. $h(-x)=(-x)f(-x)+g(-x)$
$=-xf(x)-g(x)$
$=-(xf(x)+g(x))$
$=-h(x)$, $h(x)$ is odd

3. $h(-x)=f(-x)\cdot g(-x)$
$=f(x)\cdot -g(x)$
$=-f(x)\cdot g(x)$
$=-h(x)$, $h(x)$ is odd

4. $h(-x)=((-x)^2+1)f(-x)+g(-x)$
$=(x^2+1)f(x)-g(x)$
$=-(-(x^2+1)f(x)+g(x))$
$\neq -h(x)$, $h(x)$ is neither even nor even

43. Find g if $f(x)=|x|$ and $(fg)(x)=|x||2x-5|$

solution:

$(fg)(x)=f(x)g(x)=|x||2x-5|$

then $g(x)=|2x-5|$

44. Suppose f is defined on $[0,4]$ and $g(x)=f(x+3)$. What is the domain of g ?

solution:

Domain of $f(x+3)$ is $[0,4]$

$\Rightarrow \quad 0\le x+3\le 4$ subtract 3

$-3\le x\le 1$

Domain of $g(x)$ is $[-3,1]$

45. For which functions f is there a function g such that $f=g^2$

solution:

All functions f such that $f(x)\ge 0$ for all x in the domain of f

functions with range in $[0,\infty)$

47. For which functions f is there a function g such that $f=\dfrac{1}{g}$

solution:

All function f such that $f(x)\ne 0$ for all x in the domain of f

48. Let f and g be even functions . Show that $f+g$ and $f\cdot g$ are even function.

solution:

f is even $\Rightarrow f(-x)=f(x)$

g is even $\Rightarrow g(-x)=g(x)$

* $(f+g)(-x)=f(-x)+g(-x)$

$=f(x)+g(x)$

$=(f+g)(x)$

then $f+g$ is even

* $(f\cdot g)(-x)=f(-x)\cdot g(-x)$

$=f(x)\cdot g(x)$

$=(f\cdot g)(x)$

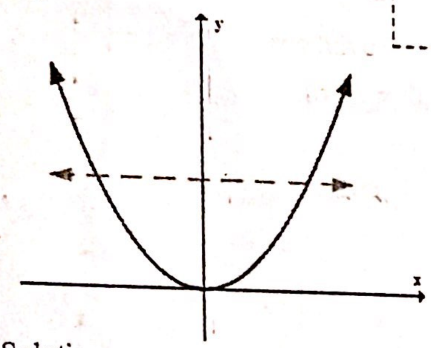
then $f\cdot g$ is even

Section (3 – 6) : INVERSE FUNCTIONS

Exercises 1 – 5 : Using the horizontal – line test , determine whether the function is one – to – one.

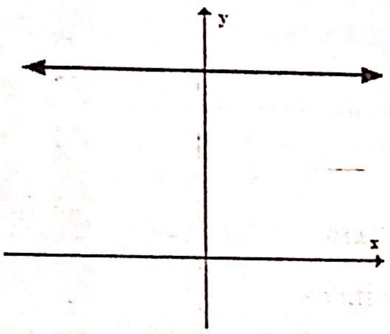
1.

- اذا قطع الخط الأفقي الرسمة في نقطة واحدة فانها one-to-one
- اذا قطع الخط الأفقي الرسمة في اكثر من نقطة فانها not one-to-one



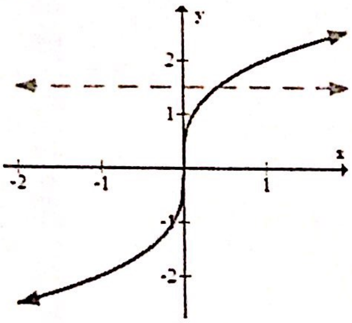
Solution:
The graph is not one – to – one , the horizontal line intersects the graph in more than one point.

2.



Solution:
The graph is not one – to – one , the horizontal line intersects the graph in more than one point.

3.



Solution:
The graph is one – to – one , the horizontal line intersects the graph in only one point.

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Exercises 6 – 11 . Assume the following functions are one – to – one . Find their inverse at the specified values

6. If $f(4) = 3$, find $f^{-1}(3)$ solution: $f^{-1}(3) = 4$
7. If $f(2) = 4$, find $f^{-1}(4)$ solution: $f^{-1}(4) = 2$
9. Find $(f \circ f^{-1})(-5)$ solution: $(f \circ f^{-1})(-5) = -5$
11. Find $(f \circ f^{-1})(0)$ solution: $(f \circ f^{-1})(0) = 0$

Exercises 12 - 16 , Prove that f and g are inverses of each other

12. $f(x) = 3x$, $g(x) = \frac{x}{3}$

solution:

$$* \ (f \circ g)(x) = f(g(x)) = f\left(\frac{x}{3}\right) = 3\left(\frac{x}{3}\right) = x$$

$$* \ (g \circ f)(x) = g(f(x)) = g(3x) = \frac{3x}{3} = x$$

then f and g are inverses of each other

لاثبات أن f و g معكوس لبعض نثبت أن

- $(f \circ g)(x) = x$
- $(g \circ f)(x) = x$

15. $f(x)=\sqrt{6-x}$, $g(x)=6-x^2$, $0\le x\le 6$

solution:

* $(f\circ g)(x)=f(g(x))$

$$=f(6-x^2)$$

$$=\sqrt{6-(6-x^2)}$$

$$=\sqrt{6-6+x^2}$$

$$=\sqrt{x^2}$$

$$=|x|=x \qquad , \quad 0\le x\le 6$$

* $(g\circ f)=g(f(x))$

$$=g(\sqrt{6-x})$$

$$=6-(\sqrt{6-x})^2$$

$$=6-(6-x)$$

$$=6-6+x=x$$

then f and g are inverses of each other

16. $f(x)=x^3-3$, $g(x)=\sqrt[3]{x+3}$

solution:

* $(f\circ g)(x)=f(g(x))$

$$=f(\sqrt[3]{x+3})$$

$$=(\sqrt[3]{x+3})^3-3$$

$$=x+3-3=x$$

* $(g\circ f)(x)=g(f(x))$

$$=g(x^3-3)$$

$$=\sqrt[3]{x^3-3+3}$$

$$=\sqrt[3]{x^3}=x$$

then f and g are inverses of each other

Exercises 23 – 33 . Determine whether the given function are one – to – one .If it is one – to – one , find its inverse

23. $f = \{(12,2),(15,4),(19,-1),(25,6),(78,0)\}$

solution:

f is one - to - one

$f^{-1} = \{(2,12),(4,15),(-1,19),(6,25),(0,78)\}$

لا يوجد أي عنصر مكرر في range
أي كل عنصر من المجال له صورة
واحدة فقط بالمدى

---

25. $h(x) = x^2 + 2$

solution:

Suppose $x_1, x_2 \in \mathbb{R}$.

$$h(x_1) = h(x_2)$$
$$x_1^2 + 2 = x_2^2 + 2$$
$$x_1^2 = x_2^2$$
$$x_1 = \pm x_2$$

then $h(x)$ is not one - to - one , $h(x)$ has no inverse

لاثبات ان الدالة $1-1$
- نفرض أن $f(x_1) = f(x_2)$ و نبسط المعادلة

-1 اذا كان الناتج $x_1 = x_2$ فان الدالة $1-1$
-2 اذا كان الناتج $x_1 = \pm x_2$ فان الدالة ليست $1-1$
-3 اذا كان x_1 لها أكثر من حل بدلالة x_2 ليست 1-1

---

28. $K(x) = |5x - 4|$

solution:

Suppose $x_1, x_2 \in \mathbb{R}$.

$$K(x_1) = K(x_2)$$
$$|5x_1 - 4| = |5x_2 - 4|$$
$$5x_1 - 4 = \pm(5x_2 - 4)$$

$5x_1 - 4 = 5x_2 - 4$ or $5x_1 - 4 = -(5x_2 - 4)$

$5x_1 = 5x_2$ or $5x_1 - 4 = -5x_2 + 4$

$x_1 = x_2$ or $5x_1 = -5x_2 + 8$

$x_1 = x_2$ or $x_1 = -x_2 + \frac{8}{5}$

لاحظ x_1 لها أكثر من حل بدلالة x_2 ليست 1-1

then $K(x)$ is not one - to - one , K has no inverse

30. $f(x)=\sqrt{x+5}$, $x \ge -5$

solution:

Suppose $x_1, x_2 \in \mathbb{R}$

$$f(x_1)=f(x_2)$$

$$\sqrt{x_1+5}=\sqrt{x_2+5} \qquad \text{(square both sides)}$$

$$x_1+5=x_2+5$$

$$x_1=x_2$$

f is one - to - one and has inverse

* To find the inverse

لايجاد معكوس الدالة

1- نستبدل $f(x)$ بـ y

2- نحل المعادلة بالنسبة لـ x

3- نضع $f^{-1}(y)$ بدلا من x

4- نستبدل كل من y بـ x والعكس

$$f(x)=\sqrt{x+5}$$

$$y=\sqrt{x+5}$$ تربيع الطرفين

$$y^2=x+5$$

$$y^2-5=x$$

$$f^{-1}(y)=y^2-5$$

$$f^{-1}(x)=x^2-5$$

32. $g(x)=\sqrt[3]{x}+4$

solution:

Suppose $x_1, x_2 \in \mathbb{R}$

$$g(x_1)=g(x_2)$$

$$\sqrt[3]{x_1}+4=\sqrt[3]{x_2}+4$$

$$\sqrt[3]{x_1}=\sqrt[3]{x_2}$$ تكعيب الطرفين

$$x_1=x_2$$

f is one - to - one , and has inverse

* To find the inverse

$$g(x)=\sqrt[3]{x}+4$$

$$y=\sqrt[3]{x}+4$$

$$y-4=\sqrt[3]{x}$$ تكعيب الطرفين

$$(y-4)^3=x \quad \Rightarrow f^{-1}(y)=(y-4)^3$$

The inverse is $f^{-1}(x)=(x-4)^3$

33. $f(x) = x\sqrt{9-x^2}$, $x \in [-3,3]$

لاحظ أن العنصرين 3 و -3 لهما نفس القيمة $f(3) = f(-3)$
فإن الدالة ليست $1-1$

solution:

$f(-3) = -3\sqrt{9-(-3)^2} = -3\sqrt{0} = 0$

$f(3) = 3\sqrt{9-(3)^2} = 3\sqrt{0} = 0$

we have $-3 \neq 3$ but $f(-3) = f(3)$

then f is not one - to - one and has no inverse

Exercises 34-39: Determine whether each pair of the following functions are inverses of each other

34. $g(x) = -x^3 - 3$, $f(x) = \sqrt[3]{-x^3 - 3}$

solution:

$$* \ (f \circ g)(x) = f(g(x))$$
$$= f(-x^3 - 3)$$
$$= \sqrt[3]{-(-x^3-3)^3 - 3} \neq x$$

f and g are not inverses to each other

35. $h(x) = \dfrac{x-1}{2}$, $r(x) = 2x + 1$

solution:

$$* \ (h \circ r)(x) = h(r(x))$$
$$= h(2x+1)$$
$$= \frac{(2x+1)-1}{2}$$
$$= \frac{2x}{2} = x$$

$$* \ (r \circ h)(x) = r(h(x))$$
$$= r\left(\tfrac{x-1}{2}\right)$$
$$= 2\left(\tfrac{x-1}{2}\right) + 1$$
$$= (x-1) + 1 = x$$

h and r are inverses to each other

39. $a(x)=\sqrt{\frac{x}{x+1}}$, $b(x)=\left(\frac{x+1}{x}\right)^2$

solution:

* $(a\circ b)(x)=a(b(x))$

$$=a\left(\left(\tfrac{x+1}{2}\right)^2\right)$$

$$=\sqrt{\frac{\left(\frac{x+1}{2}\right)^2}{\left(\frac{x+1}{2}\right)^2+1}}\neq x$$

a and b are not inverse to each other

Exercises 40 – 50 . Find the inverse of each the following functions (Assume they are 1 – 1)

41. $f(x)=-7x+11$

solution:

$$y=-7x+11$$

$$y-11=-7x$$

$$\frac{y-11}{-7}=x \quad\Rightarrow\quad f^{-1}(y)=\frac{y-11}{-7}$$

$$f^{-1}(x)=\frac{x-11}{-7}$$

43. $f(x)=-x^2+2$, $x\geq 0$

solution:

$$y=-x^2+2$$

$$y-2=-x^2 \qquad \text{(multiply by } (-1) \text{)}$$

$$-y+2=x^2$$

$$\sqrt{-y+2}=x \quad\Rightarrow\quad f^{-1}(y)=\sqrt{-y+2}$$

$$f^{-1}(x)=\sqrt{-x+2}$$

44. $f(x)=\dfrac{2}{x}$, $x \neq 0$

solution:

$$y=\frac{2}{x}$$

$$xy=2$$ ضرب طرفين = ضرب الوسطين

$$x=\frac{2}{y} \Rightarrow f^{-1}(y)=\frac{2}{y}$$

$$f^{-1}(x)=\frac{2}{x}$$

45. $f(x)=\dfrac{x+1}{x-1}$, $x \neq 1$

solution:

$$y=\frac{x+1}{x-1}$$

$$(x+1)y=x+1$$ ضرب طرفين = ضرب الوسطين

$$xy+y=x+1$$ نضع حدود x بطرف و أخذ x عامل مشترك

$$xy-x=-y+1$$

$$x(y-1)=-y+1$$

$$x=\frac{-y+1}{y-1} \Rightarrow f^{-1}(y)=\frac{-y+1}{y-1}$$

$$f^{-1}(x)=\frac{-x+1}{x-1}$$

47. $f(x)=(x+1)^{1/3}$

solution:

$$y=(x+1)^{1/3}$$

$$y^3=x+1$$ تكعيب الطرفين

$$y^3-1=x \Rightarrow f^{-1}(y)=y^3-1$$

$$f^{-1}(x)=x^3-1$$

48. $f(x) = 3x - 9$

solution:

$$y = 3x - 9$$
$$y + 9 = 3x$$
$$\frac{y+9}{3} = x \qquad \Rightarrow f^{-1}(y) = \frac{y+9}{3}$$
$$f^{-1}(x) = \frac{x+9}{3}$$

Increasing and Decreasing Function:

* f is increasing on I : If $x_1 < x_2 \quad \Rightarrow \quad f(x_1) < f(x_2)$

* f is decreasing on I : If $x_1 < x_2 \quad \Rightarrow \quad f(x_1) > f(x_2)$

اثبات التزايد او التناقص

-1 نفرض $x_1 < x_2$ اذا اثبتنا أن $f(x_1) < f(x_2)$ فان الدالة تزايديه على الفترة

-2 نفرض $x_1 < x_2$ اذا اثبتنا أن $f(x_1) > f(x_2)$ فان الدالة تناقصية على الفترة

هام جدا :

-1 المعادلة الخطية تكون اما تزايدية كلها على الأعداد الحقيقة أو تناقصية كليا

-2 المعادلة التربيعية : لابد من تقسيم الحل الى فترتين $(0,\infty)$ و $(-\infty,0)$

Exercises 51 – 57 , Determine the intervals on which each of the following functions are increasing and the intervals on which they are decreasing

51. $f(x) = 2x - 7$

solution:

لاحظ أن المعادلة خطية اما كليا تزايدية أو تناقصية

Assume $\quad x_1 < x_2 \qquad$ multiply by 2

$2x_1 < 2x_2 \qquad$ add -7 to both sides

$2x_1 - 7 < 2x_2 - 7$

Thus $\quad f(x_1) < f(x_2)$

f is increasing on $(-\infty, \infty)$

همسة في اذنك:

-1 اذا كان معامل x اشارته موجبة فانها تزايدية على الأعداد الحقيقية

-2 اذا كان معامل x اشارته سالبة فانها تناقصية على الأعداد الحقيقية

52. $f(x) = 1 - 3x$

solution:

Assume $\quad x_1 < x_2$

$-3x_1 > -3x_2$

$1 - 3x_1 > 1 - 3x_2$

Thus $\quad f(x_1) > f(x_2)$

f is decreasing on $(-\infty, \infty)$

عند الضرب بسالب نغير علامة المتباينة

54. $f(x) = x^2 - 8$

solution:

* First : If $x_1, x_2 \in [0, \infty)$

Assume $\quad x_1 < x_2 \quad$ square both sides

$x_1^2 < x_2^2$

Thus $\quad f(x_1) < f(x_2)$

f is increasing on $[0, \infty)$

* Second : If $x_1, x_2 \in (-\infty, 0]$

Assume $\quad x_1 < x_2$

$x_1^2 > x_2^2$

Thus $\quad f(x_1) > f(x_2)$

f is decreasing on $(-\infty, 0]$

عند تربيع عددين سالبين نغير علامة المتباينة
فمثلا : نعرف أن $-4 < -3$
فان $(-4)^2 > (-3)^2$
$16 > 9$

55. $f(x) = 2 - x^2$

solution:

* First : If $x_1, x_2 \in [0, \infty)$

Assume $\quad x_1 < x_2 \quad$ square both sides

$x_1^2 < x_2^2$

$-x_1^2 > -x_2^2$

Thus $\quad f(x_1) > f(x_2)$

f is decreasing on $[0, \infty)$

* Second : If $x_1, x_2 \in (-\infty, 0]$

Assume $\quad x_1 < x_2$

$x_1^2 > x_2^2$

$-x_1^2 < -x_2^2$

Thus $\quad f(x_1) < f(x_2)$

f is increasing on $(-\infty, 0]$

56. $f(x) = x^3$

solution:

Assume $x_1 < x_2$

$x_1^3 < x_2^3$

$f(x_1) < f(x_2)$

f is increasing on $(-\infty, \infty)$

57. $f(x) = -x^3$

solution:

Assume $x_1 < x_2$

$x_1^3 < x_2^3$

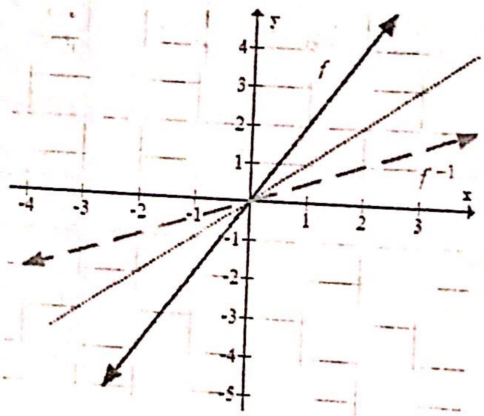
$-x_1^3 > -x_2^3$

$f(x_1) > f(x_2)$

f is decreasing on $(-\infty, \infty)$

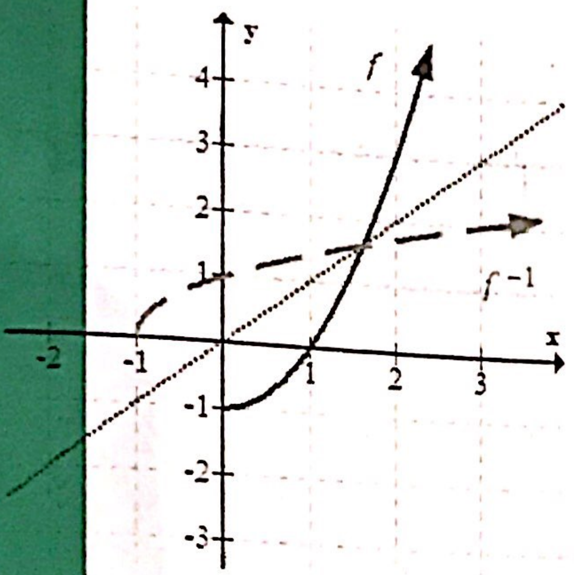
Exercises : The graph of a function f is given . on the same axis , sketch the graph Of f^{-1}

17.

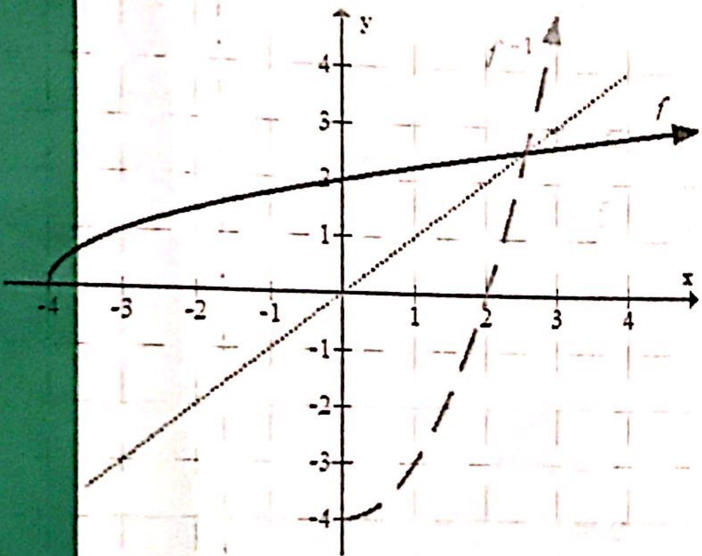


الرسمة بها الدالة f الدالة المتصلة
و يطلب رسمة الدالة العكسية f^{-1} المتماثلة معها (المقطعة)

18.



19.



ريض 140

Pre-calculus

Math - 140

Chapter 4 & 5

الجزء الثاني

شرح و حل أسئلة Exercises

مع مراجعة نهائية

0559 108 708 محمد ندا (أبو يوسف)

CHAPTER 4 : EXPONENTIAL & LOGARITHMIC FUNCTIONS
الدوال الأسية و اللوغاريتمية

SECTION (4 – 1) : EXPONENTIAL FUNCTIONS

Exercises 1 – 4 ,Which of the following function is exponential ?

1. $f(x) = 2^x$
   solution
   f is exponential to the base $a = 2$

2. $f(x) = x^3$
   solution: ليست دالة اسية لكنها دالة كثيرة حدود
   f is not exponential

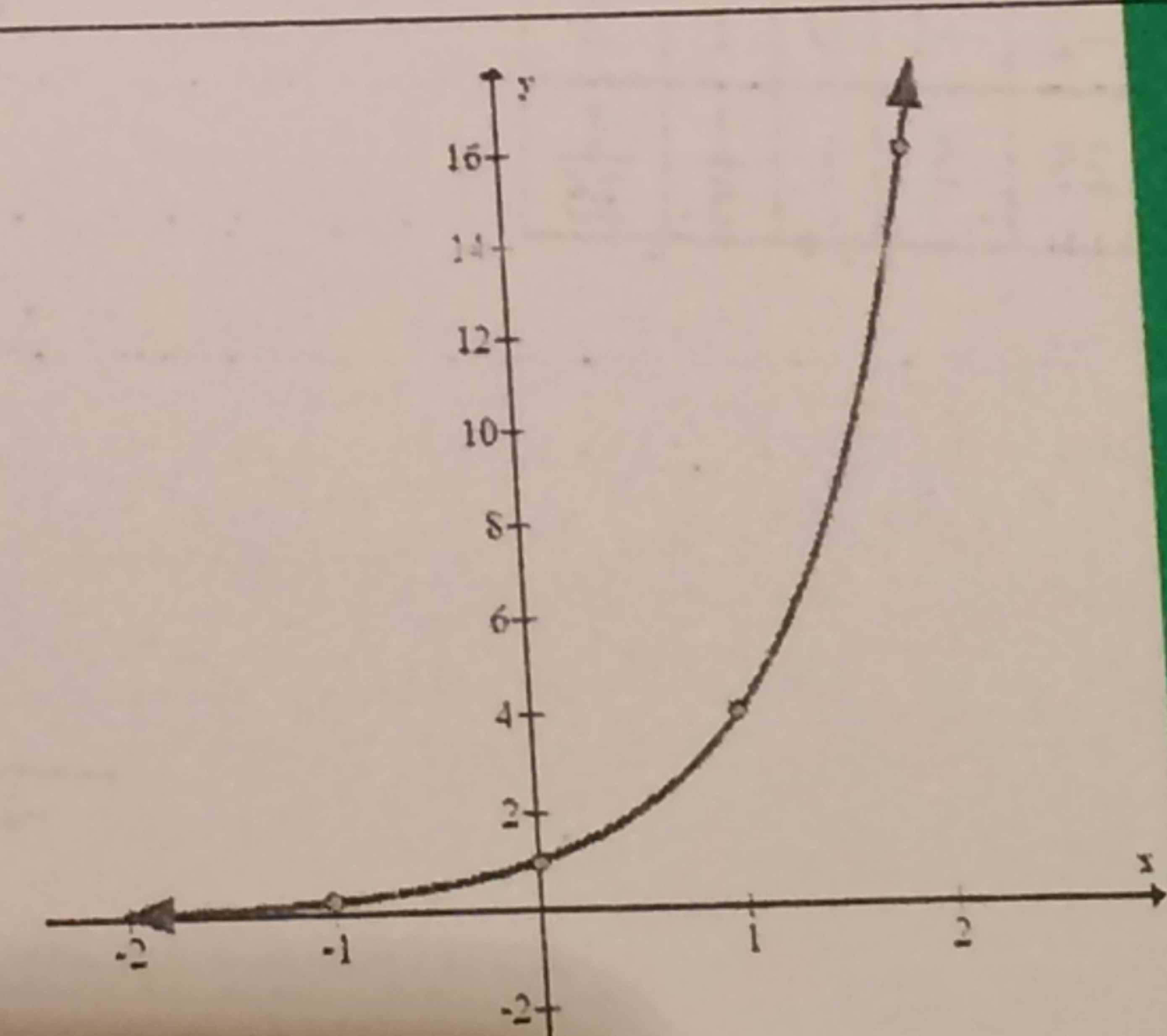
3. $f(x) = \sqrt{x}$
   solution:
   f is not exponential

4. $f(x) = \left(\sqrt{7}\right)^x$
   solution
   f is exponential to the base $a = \sqrt{7}$

Exercises 5 – 9 , Sketch the graph of each of the following exponential functions

5. $f(x) = 4^x$
   solution:

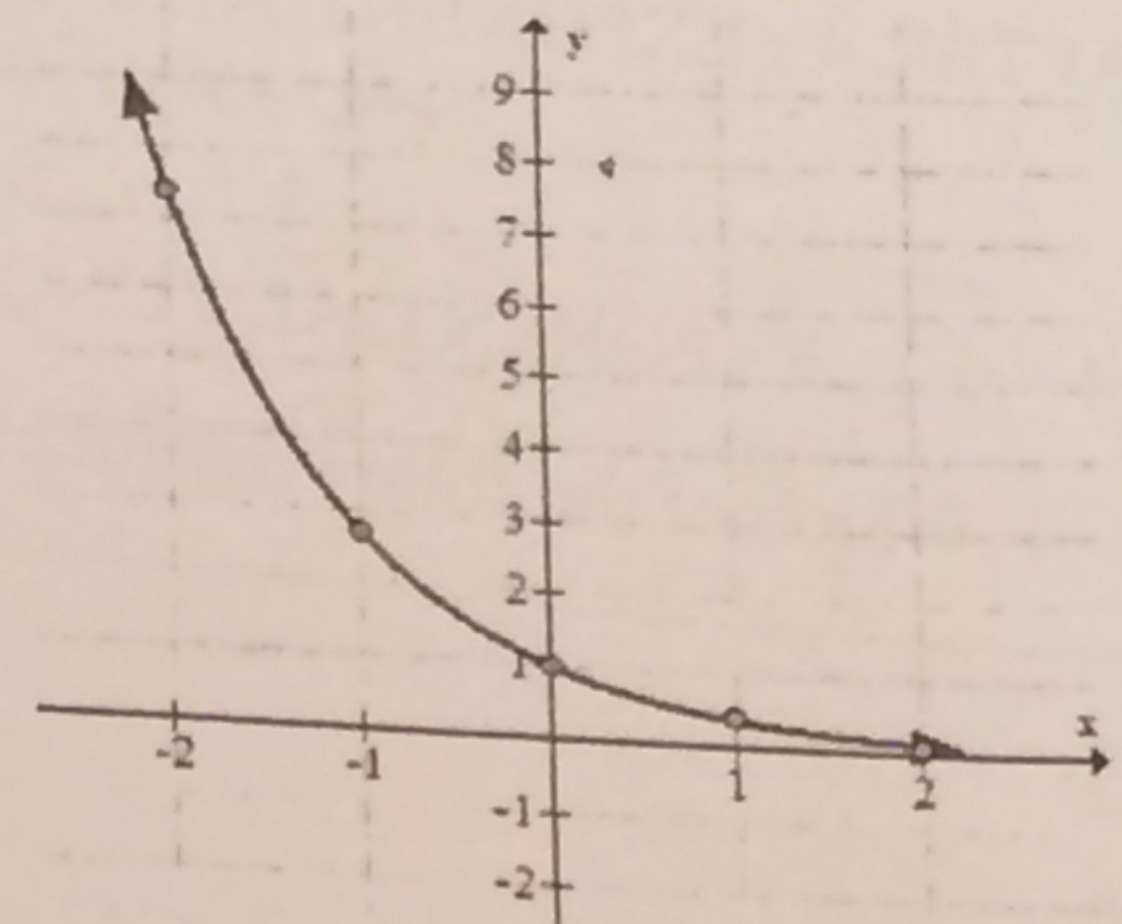
x	-2	-1	0	1	2
y	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16



6. $f(x) = e^{-x}$

solution:

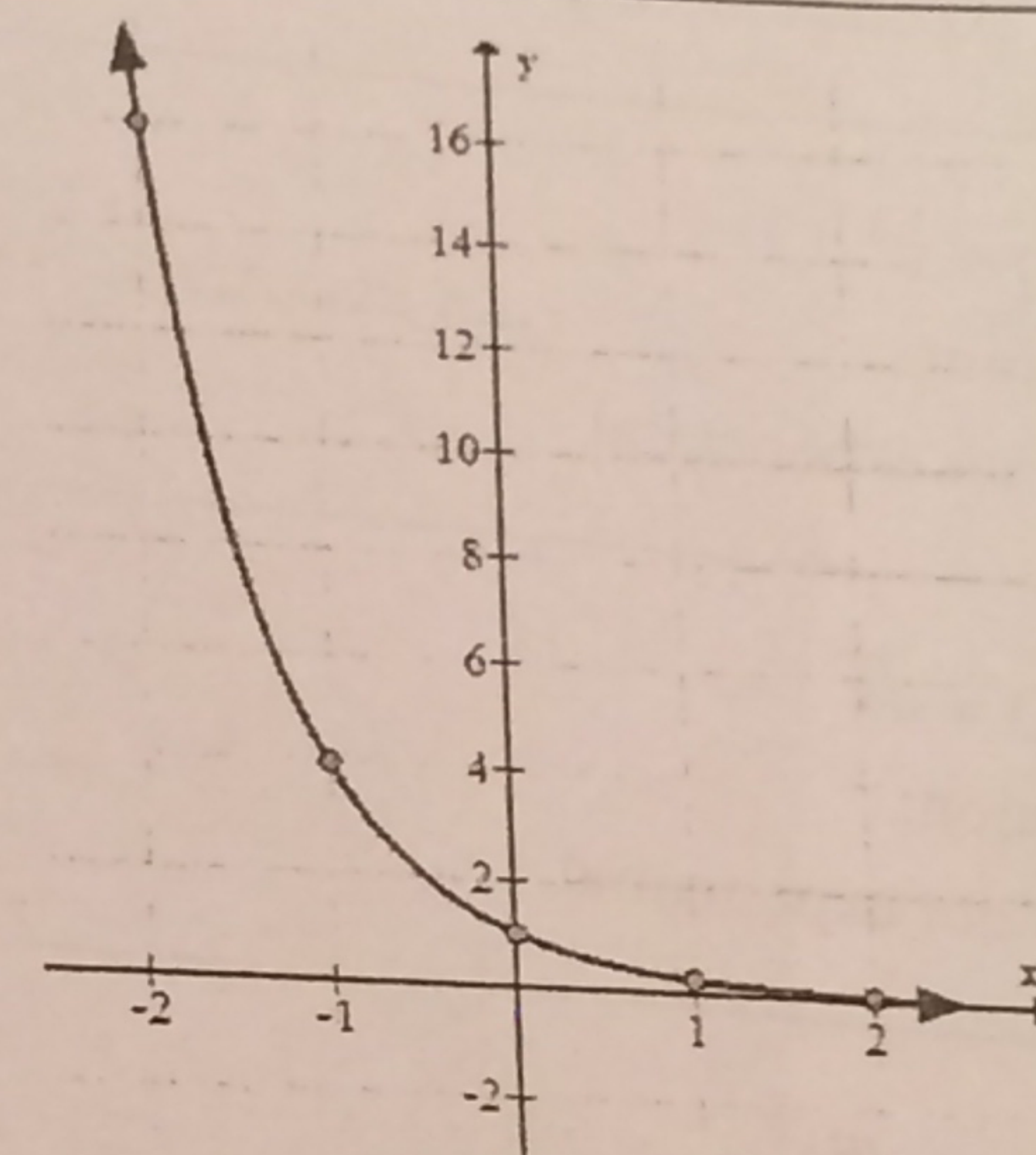
x	-2	-1	0	1	2
y	7.4	2.7	1	0.4	0.1



7. $f(x) = 4^{-x}$

solution:

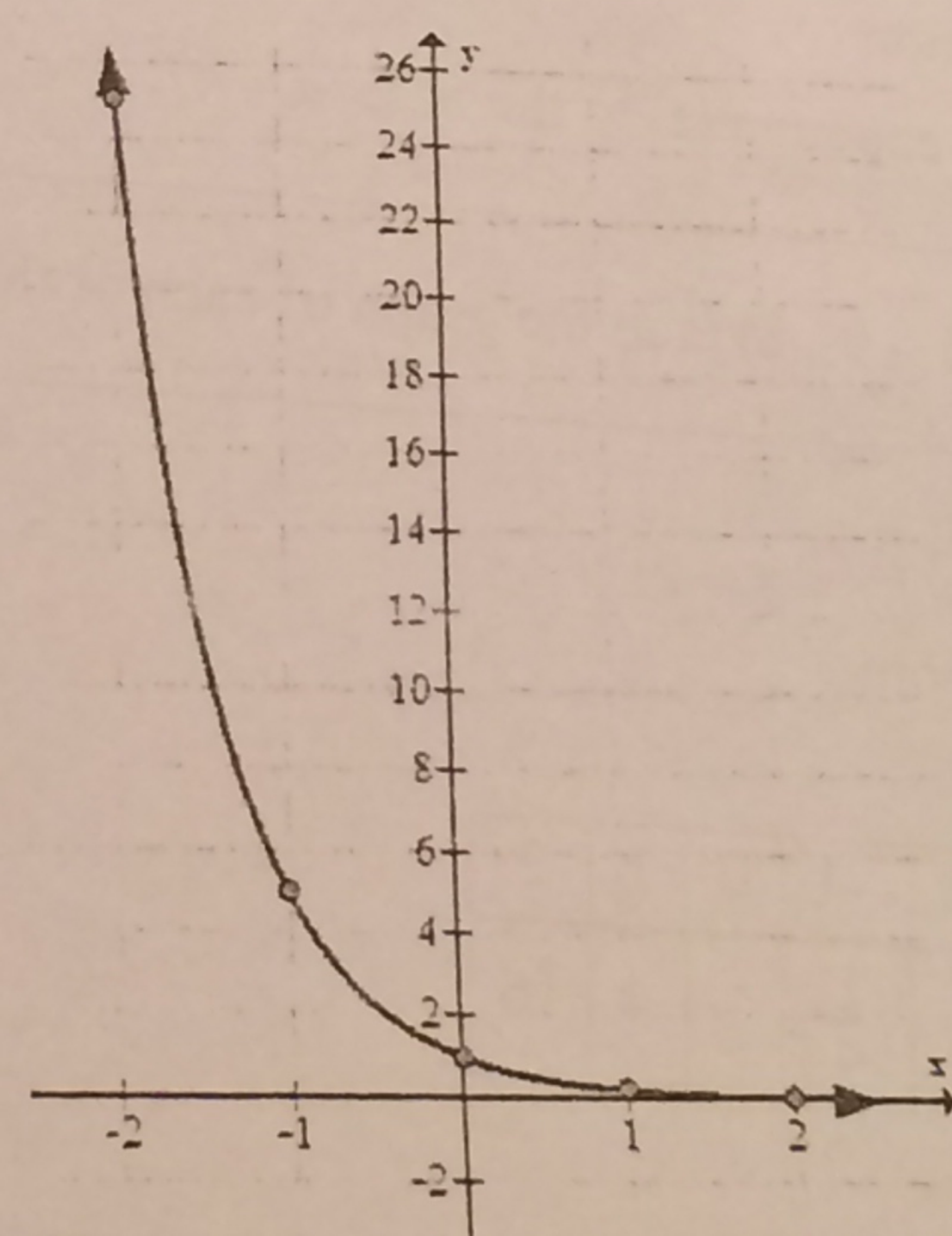
x	-2	-1	0	1	2
y	16	4	1	$\frac{1}{4}$	$\frac{1}{16}$



8. $f(x) = 5^{-x}$

solution:

x	-2	-1	0	1	2
y	25	5	1	$\frac{1}{5}$	$\frac{1}{25}$



Exercise 10 – 25 , Sketch the graph of the each of the following function using translation method

13. $f(x) = 4^x - 1$

solution:

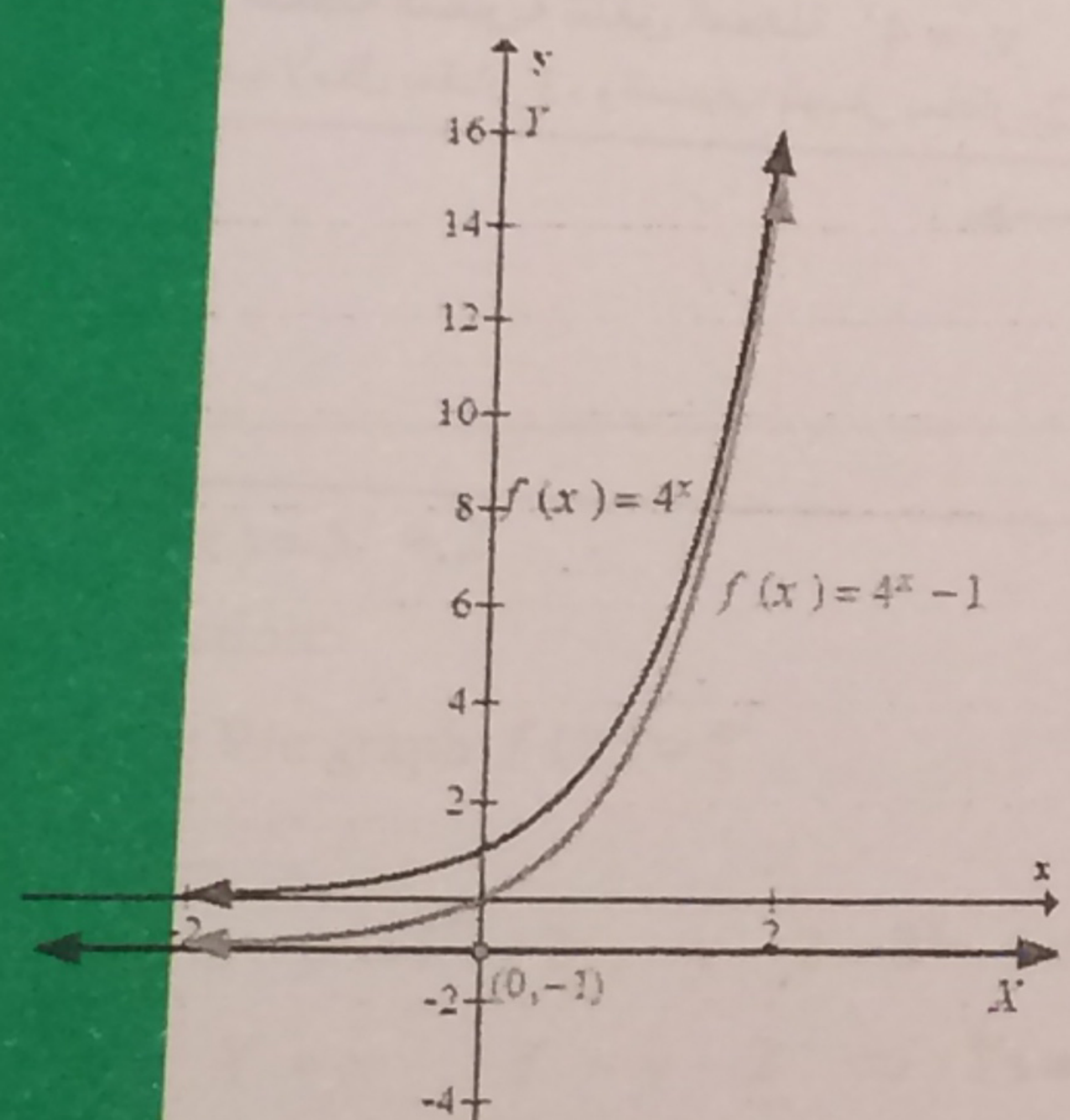
* First : We graph $f(x) = 4^x$

x	-2	-1	0	1	2
y	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16

* Second : $y = 4^x - 1$, $y + 1 = 4^x$ $\Rightarrow$ we take $h = 0$, $k = -1$

Let $X = x$, $Y = y + 1$ $\Rightarrow$ $Y = 4^X$

* The equation $Y = 4^X$ is exponential in XY – coordinate system its origin is $(h, k) = (0, -1)$



خطوات الحل:

أولا: نرسم الدالة الأم $y = 4^x$ (Exercise 5)

ثانيا: نضع المعادلة بالصيغة $y + 1 = 4^x$ لايجاد قيمة h و k مركز الاحداثيات الجديدة $(0, -1)$

ثالثا: نرسم المعادلة المطلوبة تطابق المعادلة $y = 4^x$ لكن بانسحاب لأسفل بمقدار 1

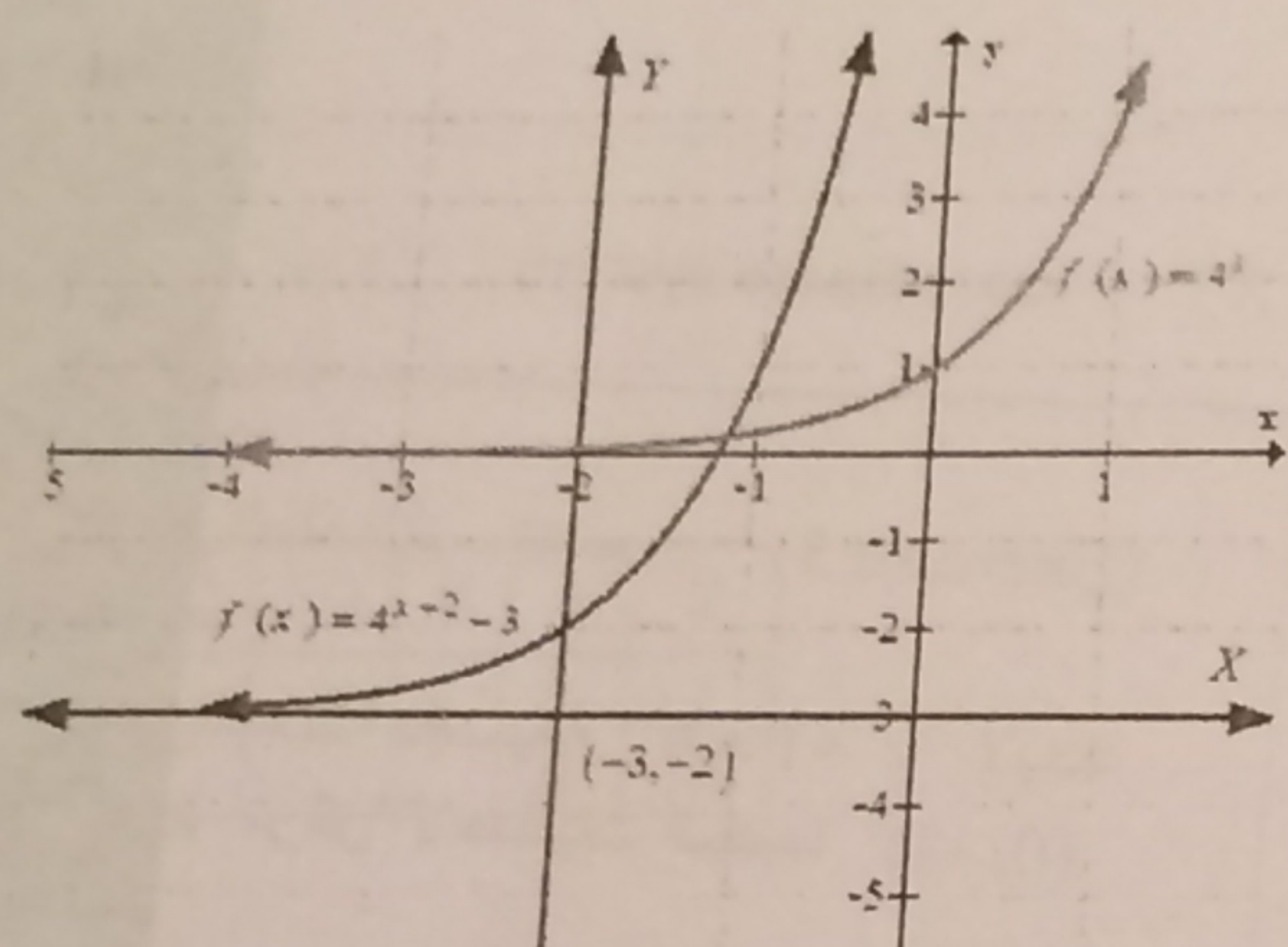
14. $f(x) = 4^{x+2} - 3$

solution:

* First : We graph $f(x) = 4^x$
* Second : $y = 4^{x+2} - 3$, $y + 3 = 4^{x+2}$ $\Rightarrow$ we take $h = -2$, $k = -3$

Let $X = x + 2$, $Y = y + 3$ $\Rightarrow$ $Y = 4^X$

* The equation $Y = 4^X$ is exponential in XY – coordinate system , its origin is $(h,k) = (-2,-3)$



خطوات الحل:

أولا: نرسم الدالة الأم $y = 4^x$ (Exercise 5)

ثانيا: نضع المعادلة بالصيغة $y + 3 = 4^{x+2}$ لايجاد قيمة h و k مركز الاحداثيات الجديدة $(-2,-3)$

ثالثا: نرسم المعادلة المطلوبة تطابق المعادلة $y = 4^x$ لكن بانسحاب لأسفل بمقدار 3- وانسحاب لليسار بمقدار 2-

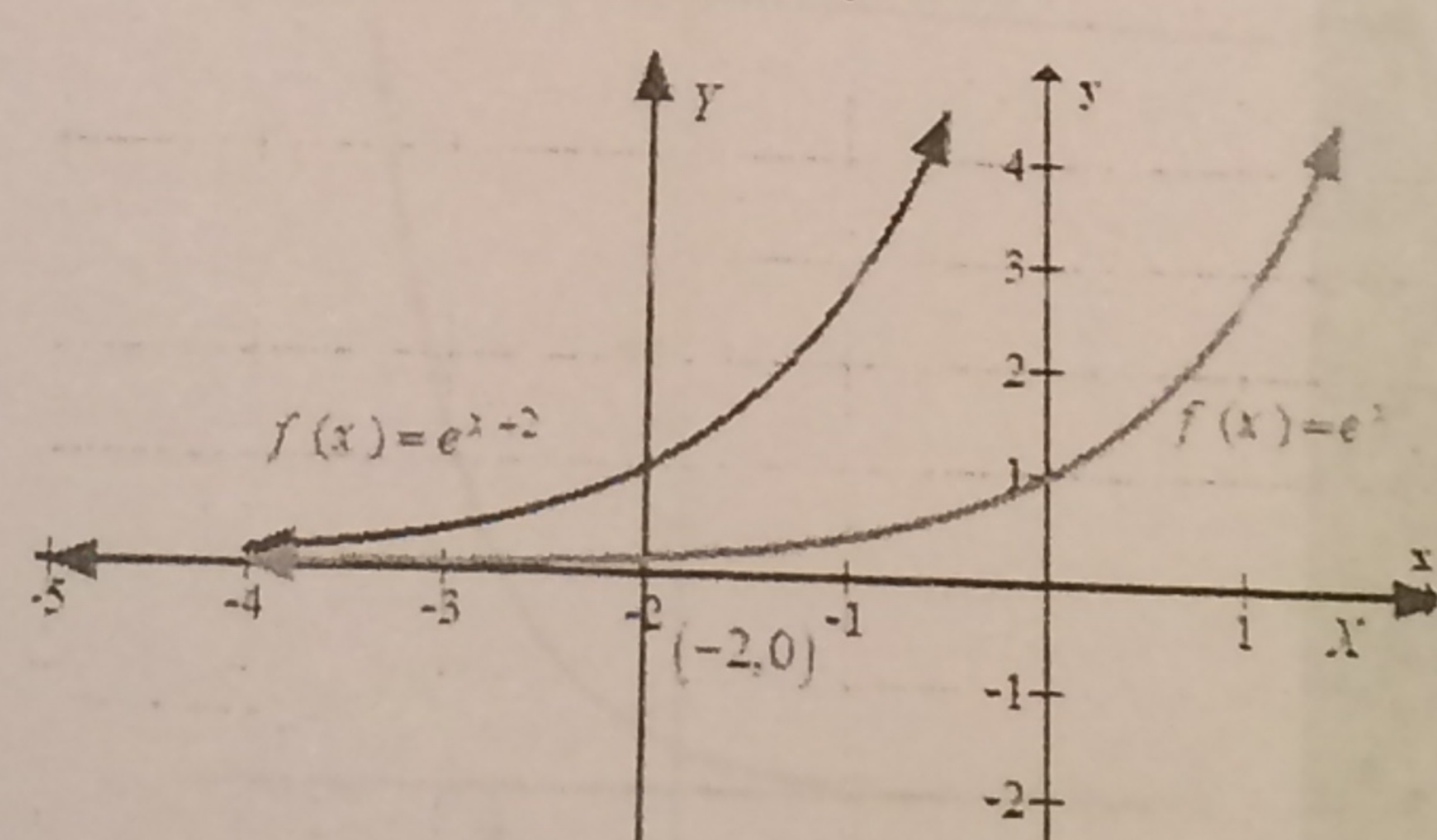
17. $f(x) = e^{x+2}$

solution:

* First : We graph $f(x) = e^x$
* Second : $y = e^{x+2}$ $\Rightarrow$ we take $h = -2$, $k = 0$

Let $X = x + 2$, $Y = y$ $\Rightarrow$ $Y = e^X$

* The equation $Y = x^X$ is exponential in XY – coordinate system , its origin is $(h,k) = (-2,0)$



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20. $f(x) = e^{x+2} - 3$

solution:

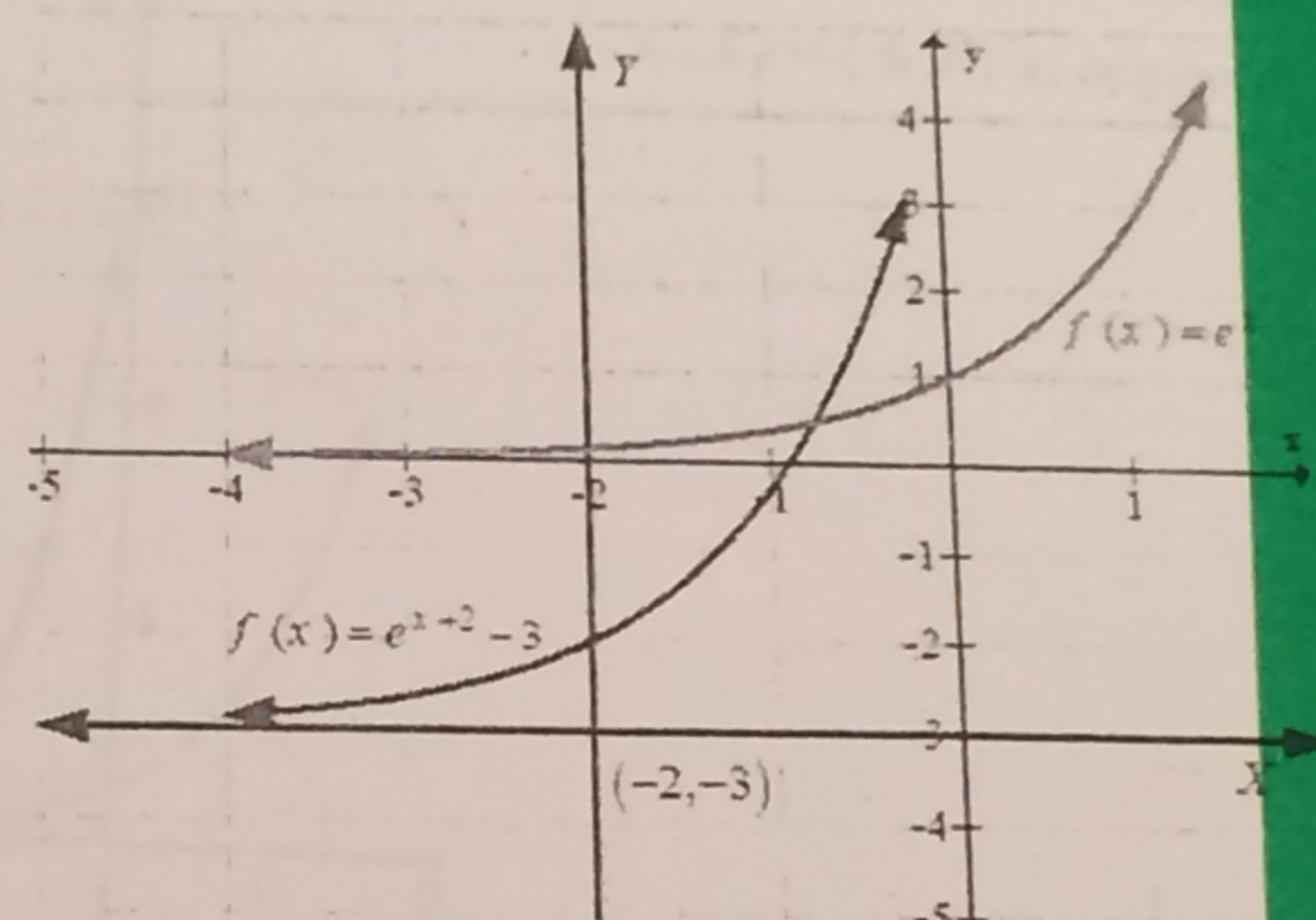
* First : We graph $f(x) = e^x$

* Second : $y = e^{x+2}$, $y + 3 = e^{x+2}$ $\Rightarrow$ we take $h = -2$, $k = -3$

Let $X = x + 2$, $Y = y + 3$ $\Rightarrow$ $Y = e^X$

* The equation $Y = x^X$ is exponential in XY – coordinate system

, its origin is $(h, k) = (-2, -3)$



22. $f(x) = 5^x + 2$

solution:

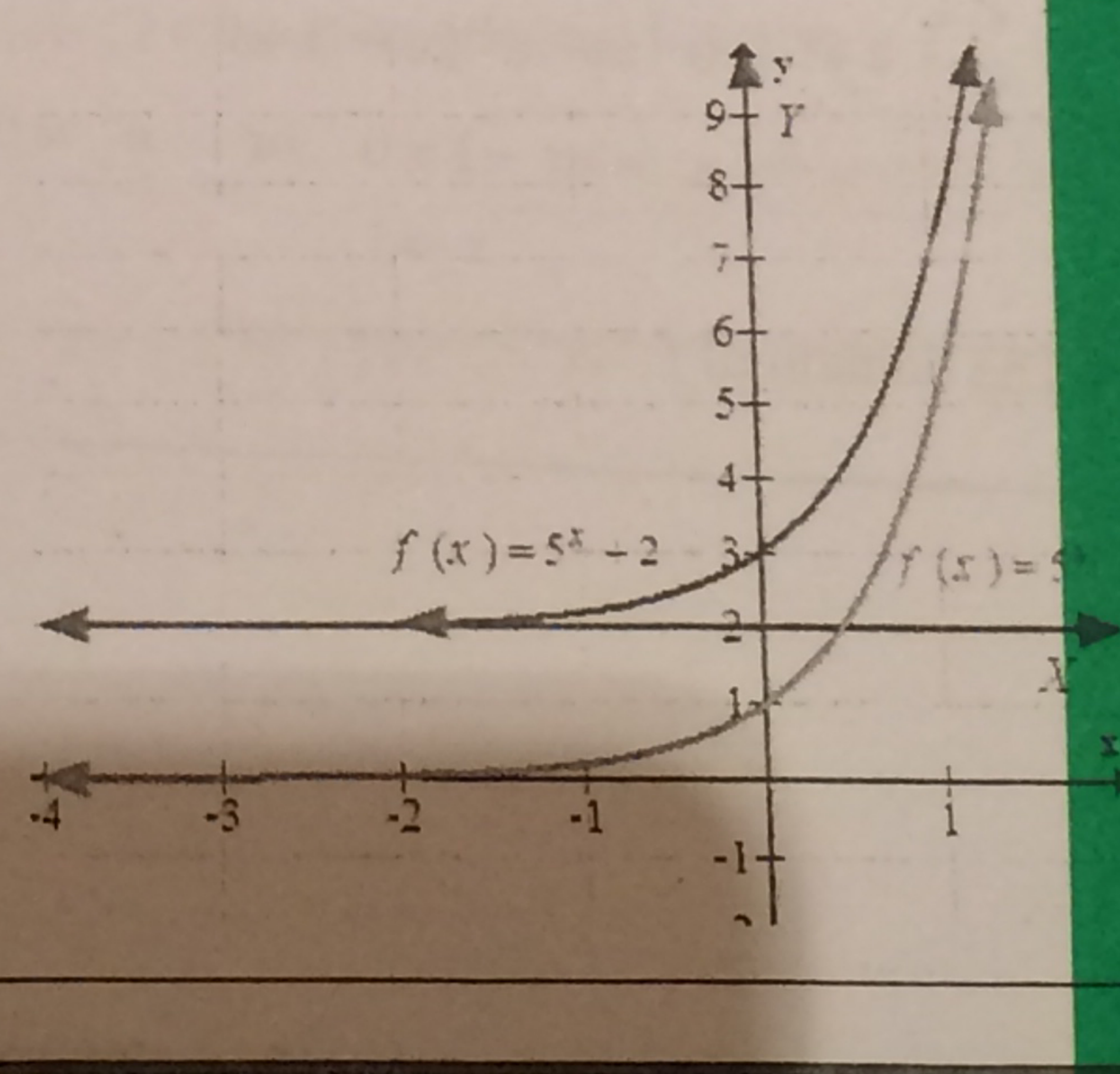
* First : We graph $f(x) = 5^x$

* Second : $y = 5^x + 2$, $y - 2 = 5^x$ $\Rightarrow$ we take $h = 0$, $k = 2$

Let $X = x$, $Y = y - 2$ $\Rightarrow$ $Y = 5^X$

* The equation $Y = 5^X$ is exponential in XY – coordinate system

its origin is $(h, k) = (0, 2)$



25. $f(x) = 4^{-x+2} - 3$

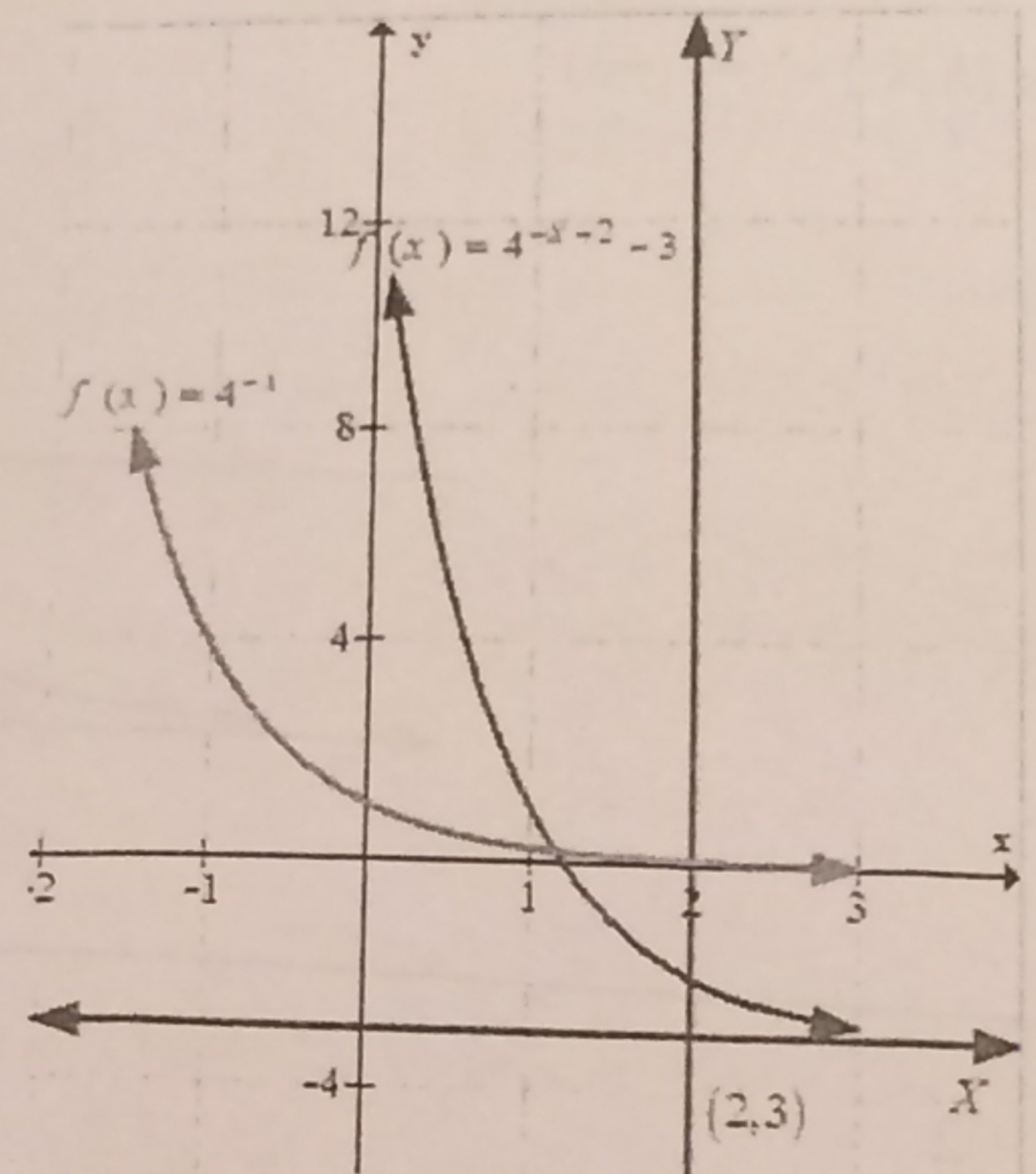
solution:

* First : We graph $f(x) = 4^{-x}$

* Second : $y = 4^{-x+2} - 3$, $y + 3 = 4^{-(x-2)}$ $\Rightarrow$ we take $h = 2$, $k = 3$

Let $X = -(x-2)$, $Y = y + 3$ $\Rightarrow$ $Y = 4^{-X}$

* The equation $Y = 4^{-X}$ is exponential in XY – coordinate system , its origin is $(h, k) = (2, 3)$



Exercises 26 – 29 , Find the domain for the following functions

مجال الدالة الكسرية : جميع الأعداد الحقيقية ما عدا أصفار المقام

26. $f(x) = \dfrac{x+2}{xe^x - e^x}$

solution:

$xe^x - e^x = 0 \Rightarrow e^x(x-1) = 0$

$x - 1 = 0$ or $e^x = 0$ has no solution (because $e^x > 0$)

$x = 1$

The domain of f is $\mathbb{R} - \{0\}$

27. $f(x) = \dfrac{5x+3}{(x^2-9)e^{2x+1}}$

solution:

$(x^2-9)e^{2x+1} = 0$

$x^2 - 9 = 0$ or $e^{2x+1} = 0$ (has no solution)

$x = \pm 3$

The domain of f is $\mathbb{R} - \{-3, 3\}$

28. $f(x) = \dfrac{2x+3}{e^{-x+3} - e^{2x-1}}$

solution:

$e^{-x+3} - e^{2x-1} = 0$

$e^{-x+3} = e^{2x-1}$

we get $-x + 3 = 2x - 1$

$-3x = -4$

$x = \dfrac{4}{3}$

The domain of f is $\mathbb{R} - \left\{\dfrac{4}{3}\right\}$

Ex : $f(x) = \dfrac{2x+3}{e^{-x+3} + 3}$

solution:

$e^{-x+3} + 3 = 0$

$e^{-x+3} = -3$ has no solution (because exponential function $e^{-x+3} > 0$)

الدالة الأسية دائما قيمتها أكبر من الصفر

Domain of f is $(-\infty, \infty)$

DEFINITIONS

1. Natural logarithmic function　　دالة اللوغاريتم ذات الأساس الطبيعي

   $f(x) = \log_e x = \ln x$

2. Common logarithmic function　　دالة اللوغاريتم ذات الأساس 10

   $f(x) = \log_{10} x = \log x$

3. Domain of $f(x) = \log_a x$ is $x \in (0, \infty)$　　where $a > 0$, $a \neq 1$

4. $y = \log_a x \quad \Leftrightarrow \quad a^y = x$

Exercises 1 – 4 , Write the given logarithmic equation in exponential form

1. $\log_3 81 = 4$

solution:

$3^4 = 81$

4. $\log \frac{1}{10000} = -4$

solution:

$10^{-4} = \frac{1}{10000}$

Ex : $\ln x = y$

solution:

$e^y = x$

Exercises 5 – 8 , Write the given exponential equation in logarithmic form

6. $\sqrt[4]{81} = 3$

solution:

$81^{\frac{1}{4}} = 3$

$\log_{81} 3 = \frac{1}{4}$

7. $7^{-2} = \frac{1}{49}$

solution

$\log_7 \frac{1}{49} = -2$

Properties of Logarithms خواص اللوغاريتمات

Property	Example
1. $\log_a 1 = 0$	$\log_4 1 = 0$, $\log 1 = 0$, $\ln 1 = 0$
2. $\log_a a = 1$	$\log_5 5 = 1$, $\log 10 = 1$, $\ln e = 1$
3. $\log_a b^r = r \log_a b$	$\log_4 6^3 = 3\log_4 6$
4. $\log_a (xy) = \log_a x + \log_b x$	$\log_3 (2)(5) = \log_3 2 + \log_3 5$
5. $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$	$\log_4 \left(\frac{7}{5}\right) = \log_4 7 - \log_4 7$
6. $\log_a x = \log_a y \Leftrightarrow x = y$	$\log_4 (x) = \log_4 (7) \Rightarrow x = 7$
7. $\log_a a^r = r$	$\log_4 4^3 = 3$
8. $a^{\log_a x} = x$	$3^{\log_3 7} = 7$
9. $\log_a x = \frac{\log_c x}{\log_c a}$	$\log_3 4 = \frac{\ln 4}{\ln 3}$

Exercises 9 – 14 , Use the properties of logarithms to find the value of the given expressions (Do not use a calculator)

9. $\log_5 125$

solution:

$\log_5 125 = \log_5 5^3 = 3$

Ex : $\ln e^{-4} - \ln e^{-5}$

solution:

* $\ln e^{-4} - \ln e^{-5} = -4 - (-5) = 1$

10. $7^{\log_7 12}$

solution:

$7^{\log_7 12} = 12$

11. $6^{2\log_6 5}$

solution:

$6^{2\log_6 5} = 6^{\log_6 5^2}$

$= 6^{\log_6 25} = 25$

* $2\log_6 5 = \log_6 5^2$

12. $\log_{1.6} 1$

solution:

$\log_{1.6} 1 = 0$

13. $3^{\log_3 4 + \log_3 5}$

solution:

$3^{\log_3 4 + \log_3 5} = 3^{\log_3 4} . 3^{\log_3 5}$

$= (4)(5) = 20$

14. $\log_6 12 + \log_6 3$

solution:

$\log_6 12 + \log_6 3 = \log_6 [(12)(3)]$

$= \log_6 36$

$= \log_6 6^2 = 2$

Ex : $\log_4 48 - \log_4 3$

solution:

$\log_4 48 - \log_4 3 = \log_4 \left(\frac{48}{3}\right)$

$= \log_4 16 = \log_4 4^2 = 2$

Domain of Logarithmic Function:

مجال الدالة اللوغاريتمية : ما داخل اللوغاريتم > صفر

Exercises : 15 – 19 , Find the domain of the given function

15. $y = \log_5(3x - 4)$

solution

$$3x - 4 > 0$$
$$3x > 4$$
$$x > \frac{4}{3}$$

Domain of the function is $\left(\frac{4}{3}, \infty\right)$

17. $g(x) = \log(x^4 + 3)$

solution:

$x^4 + 3 > 0$ for all real numbers

Domain of g is $(-\infty, \infty)$

18. $y = \log_{1.6}|2x - 7|$

solution:

$$2x - 7 = 0$$
$$2x = 7$$
$$x = \frac{7}{2}$$

Domain of the function $\mathbb{R} - \left\{\frac{7}{2}\right\}$

القيمة المطلقة تعطي قيمة موجبة و لكن
اللوغاريتم لا يقبل الصفر
و لذلك مجال اللوغاريتم للقيمة المطلقة هي
جميع الأعداد الحقيقية ما عدا اصفار الدالة

نحصل أصفار البسط و المقام و نبحث اشارة الدالة و يكون مجالها هي الفترات الموجبة
لاحظ أصفار البسط وأصفار المقام مرفوضة في المجال

19. $h(x) = \log_2 \frac{2-x}{x+3}$

solution:

The domain of the function f is the solution of the inequality $\frac{2-x}{x+3} > 0$

The zeros are $2 - x = 0$ and $x + 3 = 0 \Rightarrow x = 2$ and $x = -3$

Sign $\frac{2-x}{x+3}$ - - - - (-3) + + + + (2) - - - -

Domain $D_h = (-3, -2)$

Related problem (4)

5. $f(x) = \log_3 \sqrt{2x-4}$

solution:

$\sqrt{2x-4} > 0$

$2x - 4 > 0$

$2x > 4$

$x > 2$

Domain of f is $(2, \infty)$

Exercises 20 – 22 , Write the given logarithm in terms of ln5 and ln7 only

21. $\ln \sqrt[3]{5}$

solution:

* $\ln \sqrt[3]{5} = \ln 5^{\frac{1}{3}} = \frac{1}{3} \ln 5$

22. $\ln \frac{49}{25}$

solution:

* $\ln \frac{49}{25} = \ln 49 - \ln 25$

$= \ln 7^2 - \ln 5^2$

$= 2 \ln 7 - 2 \ln 5$

Exercises 23 – 25 , Evaluate the value of logarithm given that:

$\ln 2 = 0.69 \qquad \ln 3 = 1.10 \qquad \ln 5 = 1.61 \qquad \ln 7 = 1.95$

Find

23. $\log_3 5 = \frac{\ln 5}{\ln 3} = \frac{1.61}{1.10} = 1.46$

24. $\log_2 \frac{2}{7} = \log_2 2 - \log_2 7$

$= 1 - \frac{\ln 7}{\ln 2}$

$= 1 - \frac{1.95}{0.69} = -1.83$

25. $\log_9 18 = \log_9 [(2)(9)]$

$= \log_9 2 + \log_9 9$

$= \frac{\ln 2}{\ln 9} + 1$

$= \frac{\ln 2}{\ln 3^2} + 1$

$= \frac{\ln 2}{2\ln 3} + 1$

$= \frac{0.69}{2(1.10)} + 1 = 1.31$

Another solution

25. $\log_9 18 = \frac{\ln 18}{\ln 9}$

$= \frac{\ln[(2)(9)]}{\ln 9}$

$= \frac{\ln 2 + \ln 3^2}{\ln 3^2}$

$= \frac{\ln 2 + 2\ln 3}{2\ln 3}$

$= \frac{0.69 + 2(1.1)}{2(1.1)} = 1.31$

Realated Problem (10)

Let $a, b > 0$ such that $a \neq 1$ and $b \neq 1$. Suppose that

$\log_c 5 = 6 \quad , \quad \log_c b = 15 \quad , \quad \log_c 10 = -18$

Use the change of base property of logarithms to evaluate each of the following logarithms

1. $\log_b 25$ 2. $\log_{1/5} b$ 3. $\log b$

solution

1. $\log_b 25 = \frac{\log_c 25}{\log_c b} = \frac{\log_c 5^2}{\log_c b} = \frac{2\log_c 5}{\log_c b} = \frac{2(6)}{15} = \frac{4}{5}$

2. $\log_{1/5} b = \frac{\log_c b}{\log_c 1/5} = \frac{\log_c b}{\log_c 5^{-1}} = \frac{\log_c b}{-\log_c 5} = \frac{15}{-6} = -\frac{5}{2}$

3. $\log b = \frac{\log_c b}{\log_c 10} = \frac{15}{-18} = -\frac{5}{6}$

Exercises 26 – 29 , Expand the given logarithmic expression . Express all powers as factors

27. $\log\left(\dfrac{x^3y^7}{(y-2)^4}\right)$, $x > 0$ and $y > 2$

solution:

* $\log\left(\dfrac{x^3y^7}{(y-2)^4}\right) = \log x^3 + \log y^7 - \log(y-2)^4$

$= 3\log x + 7\log y - 4\log(y-2)$

29. $\log_3\sqrt{(x-3)\dfrac{y}{z}}$, $x > 3$, $y > 0$

solution:

* $\log_3\sqrt{(x-3)\dfrac{y}{z}} = \log_3\left[(x-3)\dfrac{y}{z}\right]^{\frac{1}{2}}$

$= \dfrac{1}{2}\log_3\left[(x-3)\dfrac{y}{z}\right]$

$= \dfrac{1}{2}\left[\log_3(x-3) + \log_3\dfrac{y}{z}\right]$

$= \dfrac{1}{2}\left[\log_3(x-3) + \log_3 y - \log_3 z\right]$

Exercises 30 – 33 , Write the given function using a single logarithm . Express the factor as a power

30. $\dfrac{1}{3}\log x + \log(x+2)$

solution:

* $\dfrac{1}{3}\log x + \log(x+2) = \log x^{\frac{1}{3}} + \log(x+2)$

$= \log\left[x^{1/3}(x+2)\right]$

$= \log\left[\sqrt[3]{x}\,(x+2)\right]$

31. $\dfrac{4}{3}\ln 8 - \ln 4$

solution :

* $\dfrac{4}{3}\ln 8 - \ln 4 = \ln 8^{\frac{4}{3}} - \ln 4$

$= \ln 16 - \ln 4$

$= \ln\dfrac{16}{4} = \ln 4$

32. $\log_2(x^2-4)-2\log_2(x+2)$

solution:

* $\log_2(x^2-4)-\log_2(x+2)^2=\log_2\dfrac{x^2-4}{(x+2)^2}$

$$=\log_2\frac{(x-2)(x+2)}{(x+2)^2}$$

$$=\log_2\frac{(x-2)}{(x+2)}$$

33. $\ln\left(\dfrac{x}{x-3}\right)+\ln\left(\dfrac{x+3}{x}\right)-\ln(x^2-9)$

solution:

* $\ln\left(\dfrac{x}{x-3}\cdot\dfrac{x+3}{x}\right)-\ln(x^2-9)=\ln\left(\dfrac{x+3}{x-3}\right)-\ln(x^2-9)$

$$=\ln\left(\frac{x+3}{x-3}\div(x^2-9)\right)$$

$$=\ln\left(\frac{x+3}{x-3}\cdot\frac{1}{x^2-9}\right)$$

$$=\ln\left(\frac{x+3}{x-3}\cdot\frac{1}{(x-3)(x+3)}\right)$$

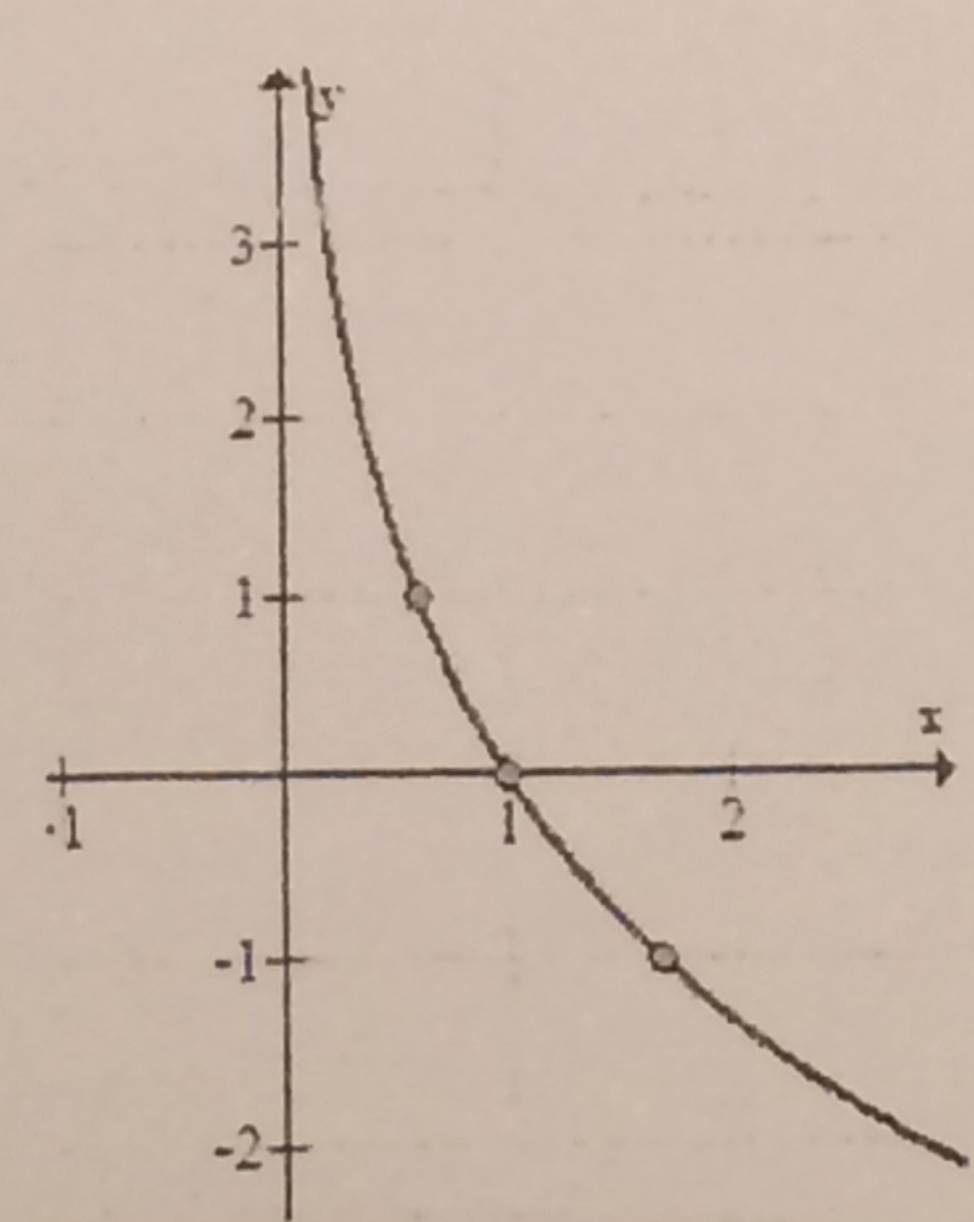
$$=\ln\left(\frac{1}{(x-3)^2}\right)=\ln(x-3)^{-2}$$

Exercises 34 – 36 , Sketch the graph of the given function

35. $y=\log_{0.6}x$

solution:

x	0.6	1	$\frac{10}{6}$
y	1	0	−1



Exercises 34 – 36 , Sketch the graph of each of the following transformation of $f(x) = \log_{1/4} x$

37. $y = \log_{1/4}(x+2)$

solution:

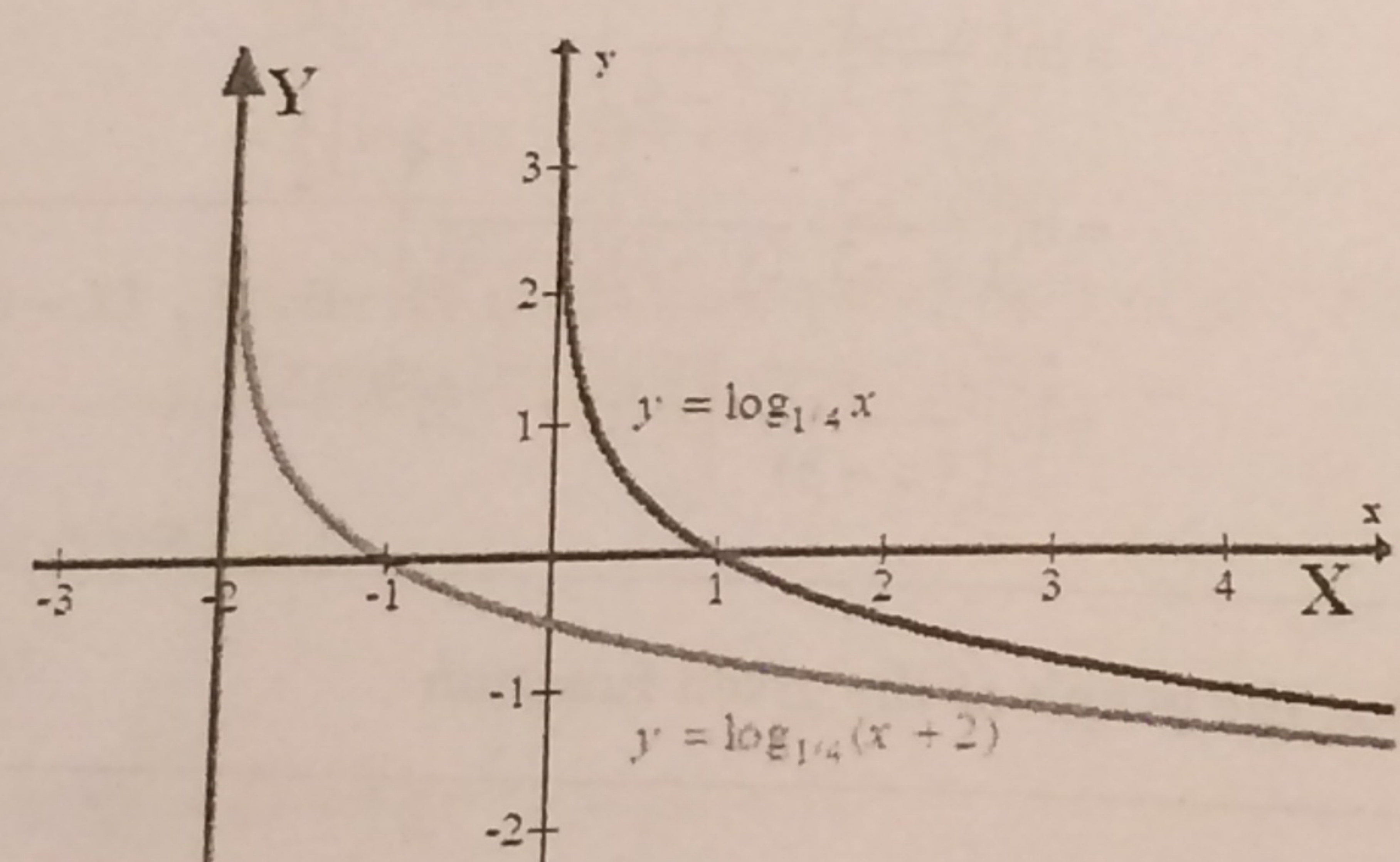
First : We graph $y = \log_{1/4} x$

x	$\frac{1}{4}$	1	4
y	1	0	-1

Second : We graph $y = \log_{1/4}(x+2)$, $h = -2$, $k = 0$

Let $X = x + 2$ and $Y = y$

Third : The equation in XY – coordinate system is $Y = \log_{1/4} X$ whose its origin is $(-2, 0)$



39. $y = \log_{1/4}(x+2) - 3$

solution:

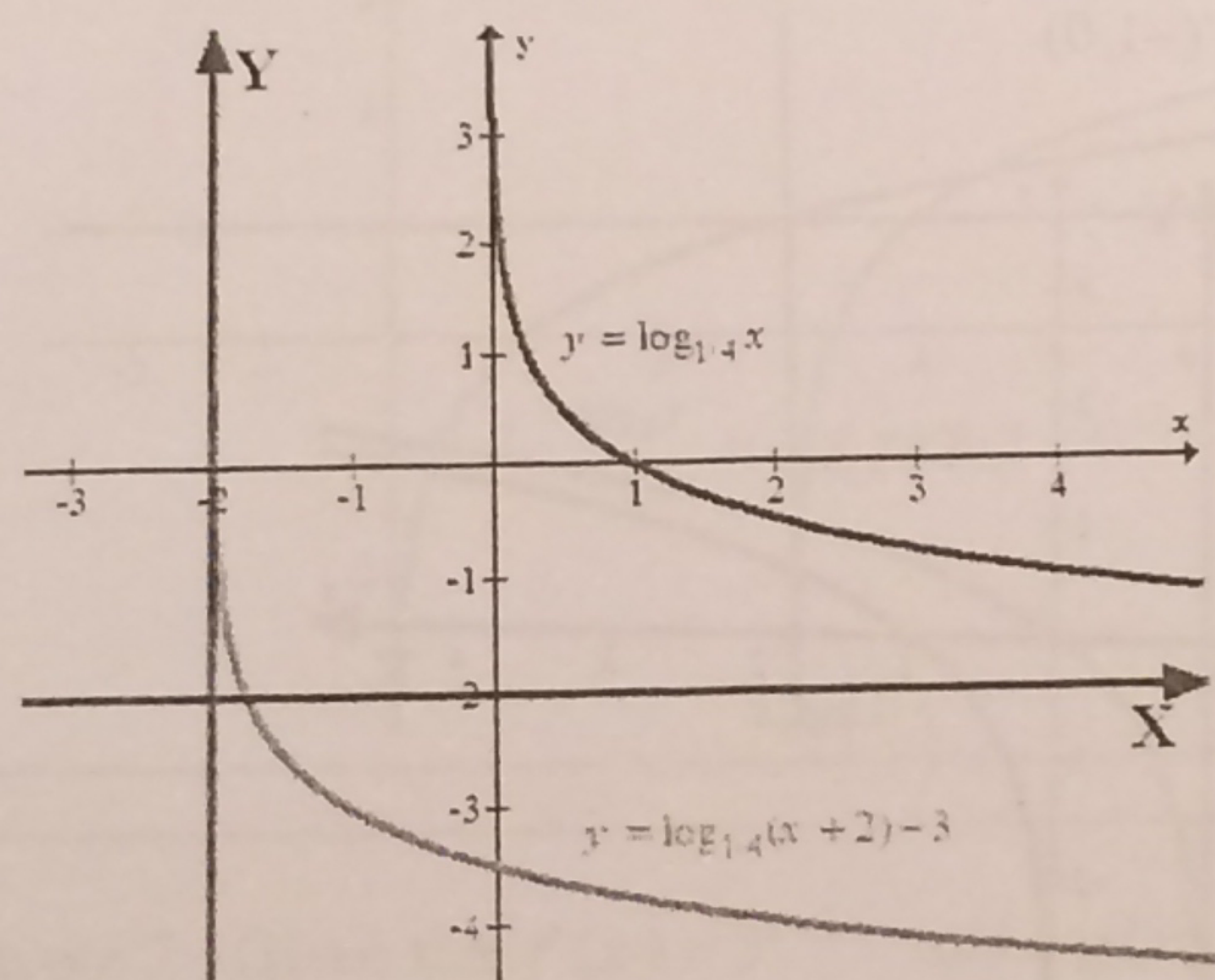
First : We graph $y = \log_{1/4} x$

x	$\frac{1}{4}$	1	4
y	1	0	-1

Second : We graph $y = \log_{1/4}(x+2) - 3$, $h = -2$, $k = -3$

Let $X = x + 2$ and $Y = y + 3$

Third : The equation in XY – coordinate system is $Y = \log_{1/4} X$ whose its origin is $(-2, -3)$



Exercises 41 – 44 , Sketch the graph of each of the following transformation of $f(x) = \log_3 x$

42. $y = \log_3(x+1)$

solution:

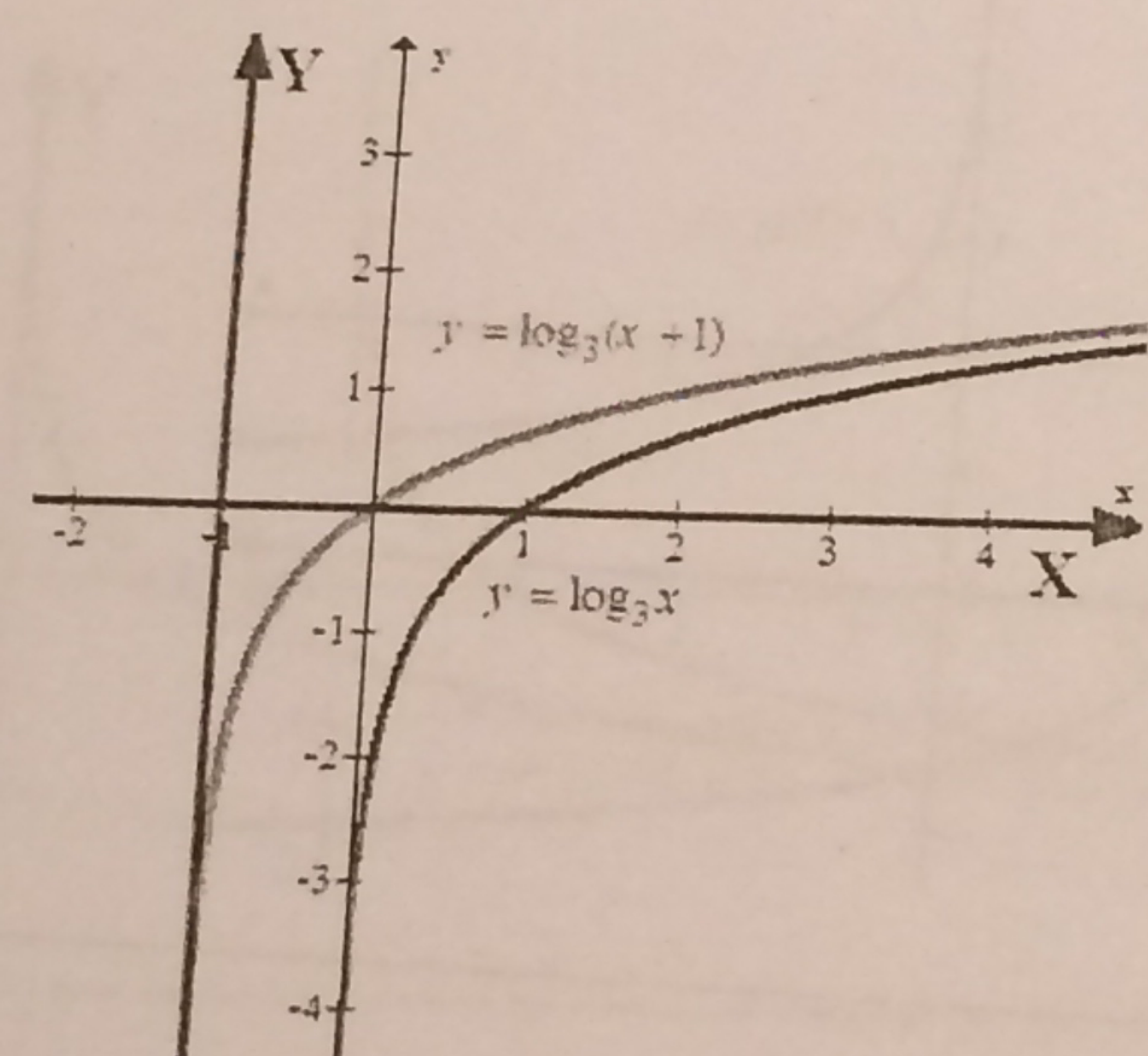
First : We graph $y = \log_3 x$

x	$\frac{1}{3}$	1	3
y	-1	0	1

Second :We graph $y = \log_3(x+1)$, $h = -1$, $k = 0$

Let $X = x + 1$ and $Y = y$

Third : The equation in XY – coordinate system is $Y = \log_3 X$ whose its origin is $(-1, 0)$



44. $y = \log_3(x-3)+1$

solution:

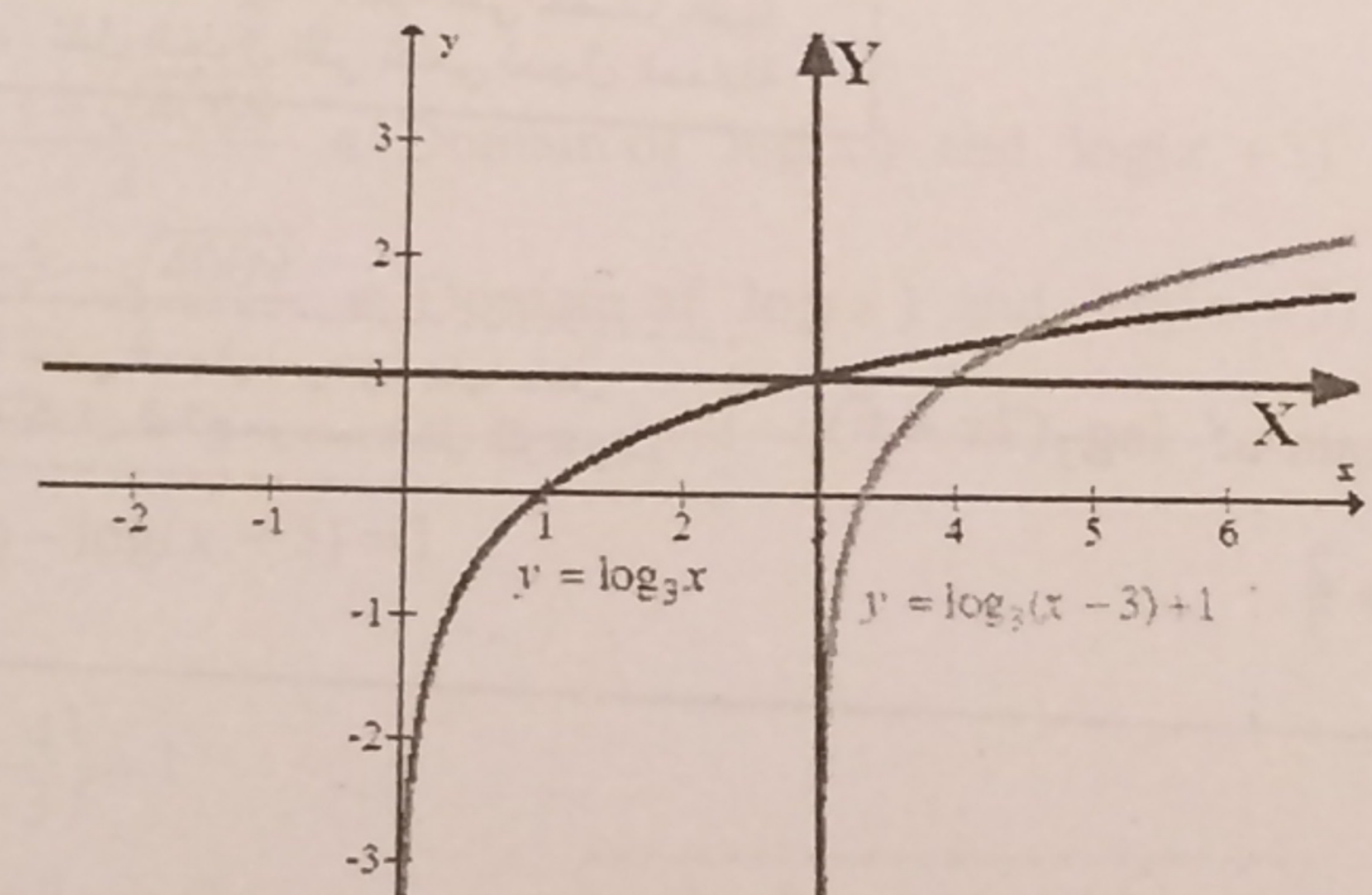
First : We graph $y = \log_3 x$

x	$\frac{1}{3}$	1	3
y	-1	0	1

Second : We graph $y = \log_3(x-3)+1$, $h=3$, $k=1$

Let $X = x-3$ and $Y = y-1$

Third : The equation in XY – coordinate system is $Y = \log_3 X$ whose its origin is (3,1)



Related Problem 7 : Given that $f(x) = 3^{x+1}$ and $g(x) = 2+\log_3 x$

Find $(g \circ f)(x)$ where $x \in \mathbb{R}$

solution:

* $(g \circ f)(x) = g(f(x))$

$= g\left(3^{x+1}\right)$

$= 2+\log_3 3^{x+1}$

$= 2+x+1 = x+3$

خطوات حل الأمثلة التالية :

1- نضع جميع الحدود التي تحتوي log في طرف
2- نستخدم خواص اللوغاريتمات لجعلها لوغاريتم واحد
3- نحول المعادلة الى معادلة اسية
4- نحل المعادلة الأسية التي حصلنا عليها
5- نقبل الحلول التي تنتمي لمجال المعادلة

Exercises 1 – 39 , Solve for x

1. $\log_2(7x+6)=3$

solution:

1- نحول المعادلة الى معادلة اسية
2- نحل المعادلة الأسية التي حصلنا عليها
3- نقبل الحلول التي تنتمي لمجال المعادلة

$7x+6=2^3$

$7x+6=8$

$7x=2$

$x=\frac{2}{7}\in$ Domain of $\log_2(7x+6)$

للتاكد أن القيمة تنتمي للمجال نعوض عنها داخل اللوغاريتم و التأكد ان الناتج موجب داخل اللوغاريتم

The solution is $x=\frac{2}{7}$

2. $\log_3(2x+4)=2$

solution:

$2x+4=3^2$

$2x=5$

$x=\frac{5}{2}\in$ Domain $\log_3(2x+4)$

The solution is $x=\frac{5}{2}$

6. $\log(x^5)=3$

solution:

$x^5=10^3$

$x^5=1000$

$x=\sqrt[5]{1000}\in$ Domain of $\log(x^5)$

The solution is $x=\sqrt[5]{1000}$

7. $\log(x) + \log(x+3) = 3$

solution:

$\log(x(x+3)) = 3$ $\boxed{\log x = \log_{10} x}$

$x(x+3) = 10^3$

$x^2 + 3x = 1000 \quad , \quad x^2 + 3x - 1000 = 0$

Using quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{3^2 - 4(1)(-1000)}}{2(1)}$$

$$= \frac{-3 \pm \sqrt{4009}}{2}$$

$x = \frac{-3 + \sqrt{4009}}{2} \in$ Domain of $\log(x)$ and $\log(x+3)$

but $x = \frac{-3 - \sqrt{4009}}{2} \notin$ Domain of $\log(x)$ and $\log(x+3)$

9. $\log(x+4) - \log(x+3) = 1$

solution:

$$\log\left(\frac{x+4}{x+3}\right) = 1$$

$\frac{x+4}{x+3} = 10^1$ ضرب الطرفين = ضرب الوسطين

$$10(x+3) = x+4$$

$$10x + 30 = x + 4$$

$$9x = -26$$

$x = -\frac{26}{9} \in$ Domain of $\log(x+4)$ and $\log(x+3)$

The solution is $x = -\frac{26}{9}$

11. $\log_6(x^2) - \log_6(x+1) = 1$

solution:

$$\log_6\left(\frac{x^2}{x+1}\right) = 1$$

$$\frac{x^2}{x+1} = 6$$

ضرب الطرفين = ضرب الوسطين

$$x^2 = 6x + 6$$

$$x^2 - 6x - 6 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(-6)}}{2(1)}$$

$$= \frac{6 \pm 2\sqrt{15}}{2} = 3 \pm \sqrt{15}$$ $\in$ Domain of $\log_6(x^2)$ and $\log_6(x+1)$

The solution is $x = 3 \pm \sqrt{15}$

---

13. $\log(x+12) = \log(x) + \log(12)$

solution:

$$\log(x+12) = \log(12x)$$

$$x + 12 = 12x$$

$12 = 11x \quad \Rightarrow x = \frac{12}{11}$ $\in$ Domain of $\log(x+12)$ and $\log(x)$

The solution is $x = \frac{12}{11}$

---

16. $\ln(x) + \ln(x-6) = \ln(6x)$

solution:

$$\ln\left(x(x-6)\right) = \ln(6x)$$

$$x^2 - 6x = 6x$$

$$x^2 - 12x = 0$$

$$x(x-12) = 0$$

$x = 0$ or $x = 12$

$x = 12 \in$ Domain of $\ln(x)$, $\ln(x-6)$ and $\ln(6x)$

$x = 0 \notin$ Domain of $\ln(x)$, $\ln(x-6)$ and $\ln(6x)$

The solution is $x = 12$

17. $\log_3 \sqrt{x+1} = 1$

solution:

$\sqrt{x+1} = 3$ square both sides

$x + 1 = 9$

$x = 8 \in$ Domain of $\sqrt{x+1}$

The solution is $x = 8$

18. $\log_2\left(2^{5x+1}\right) = 6$

solution:

$5x + 1 = 6$

$5x = 5$

$x = 1$

The solution is $x = 1$

19. $2\ln(\sqrt{x}) - \ln(x-1) = \ln 3$

solution:

$\ln(\sqrt{x})^2 - \ln(x-1) = \ln 3$

$\ln(x) - \ln(x-1) = \ln 3$

$\ln\left(\frac{x}{x-1}\right) = \ln 3$

$\frac{x}{x-1} = 3$ ضرب الطرفين = ضرب الوسطين

$3x - 3 = x$

$2x = 3 \Rightarrow x = \frac{3}{2} \in \ln(\sqrt{x})$ and $\ln(x-1)$

The solution is $x = \frac{3}{2}$

21. $\log_2(x+7) + \log_2(x+8) = 1$

solution:

$\log_2(x+7)(x+8) = 1$

$(x+7)(x+8) = 2$

$x^2 + 15x + 56 = 2$

$x^2 + 15x + 54 = 0$

$(x+6)(x+9) = 0$

$x = -6$ or $x = -9$

$x = -6 \in$ Domain of $\log_2(x+7)$ and $\log_2(x+8)$

$x = -9 \notin$ Domain of $\log_2(x+7)$ and $\log_2(x+8)$

The solution is $x = -6$

23. $\log_2(\log_3(x+1)) = 2$

solution:

$$\log_3(x+1) = 2^2$$
$$\log_3(x+1) = 4$$
$$x+1 = 3^4$$
$$x+1 = 81$$
$$x = 80 \quad \in \log_2(\log_3(x+1))$$

The solution is $x = 80$

25. $7^{4x-7} = 3^{9x-6}$

solution:

$$\ln 4^{4x-7} = \ln 3^{9x-6}$$
$$(4x-7)\ln 4 = (9x-6)\ln 3$$
$$4x \ln 7 - 7\ln 4 = 9x \ln 3 - 6\ln 3$$
$$4x \ln 7 - 9x \ln 3 = 7\ln 4 - 6\ln 3$$
$$x(4\ln 7 - 9\ln 3) = 7\ln 4 - 6\ln 3$$
$$x = \frac{7\ln 4 - 6\ln 3}{4\ln 4 - 9\ln 3}$$
$$= \frac{7\ln 2^2 - 6\ln 3}{4\ln 2^2 - 9\ln 3} = \frac{14\ln 2 - 6\ln 3}{8\ln 2 - 9\ln 3}$$

خطوات الحل:
1- نأخذ ln للطرفين
2- ننزل الأس حسب خواص اللوغاريتم
3- نضرب الأقواس
4- نضع الحدود التي تحتوي x في طرف الأيسر
5- نأخذ x عامل مشترك
6- نقسم على القوس المضروب بـ x

27. $100 - 100\left(\frac{1}{4}\right)^x = 70$

solution:

$$-100\left(\frac{1}{4}\right)^x = 70 - 100 = -30 \qquad \text{Divide by } -100$$
$$\left(\frac{1}{4}\right)^x = \frac{3}{10}$$
$$\ln\left(\frac{1}{4}\right)^x = \ln\frac{3}{10}$$
$$x\ln\left(\frac{1}{4}\right) = \ln\frac{3}{10}$$
$$x = \frac{\ln\frac{3}{10}}{\ln\frac{1}{4}}$$
$$= \frac{\ln 3 - \ln 10}{\ln 1 - \ln 4} = \frac{\ln 3 - \ln 10}{-\ln 4}$$

31. $3e^{0.09t} = e^{0.14t}$

solution

$$3 = \frac{e^{0.14t}}{e^{0.09t}}$$

$$3 = e^{0.14t - 0.09}$$

$$3 = e^{0.05t}$$

$$\ln 3 = e^{0.05t}$$

$$\ln 3 = 0.05t$$

$$\frac{\ln 3}{0.05} = t \quad \rightarrow \quad t = 20 \ln 3$$

33. $2^{x+1} = 3^{1-2x}$

solution:

$$\ln 2^{x+1} = \ln 3^{1-2x}$$

$$(x+1)\ln 2 = (1-2x)\ln 3$$

$$x \ln 2 + \ln 2 = \ln 3 - 2x \ln 3$$

$$x \ln 2 + 2x \ln 3 = \ln 3 - \ln 2$$

$$x(\ln 2 + 2\ln 3) = \ln 3 - \ln 2$$

$$x = \frac{\ln 3 - \ln 2}{\ln 2 + 2\ln 3}$$

خطوات الحل:

1- نأخذ ln للطرفين
2- ننزل الأس حسب خواص اللوغاريتم
3- نضرب الأقواس
4- نضع الحدود التي تحتوي x في طرف الأيسر
5- نأخذ x عامل مشترك
6- نقسم على القوس المضروب بـ x

36. $xe^{-x} + 2e^{-x} = 0$

solution:

$$e^{-x}(x+2) = 0$$

$e^{-x} = 0$ has no solution

or $x + 2 = 0 \Rightarrow x = -2$

The solution is $x = -2$

37. $\dfrac{e^x + e^{-x}}{2} = 2$

solution:

$e^x + e^{-x} = 4$ (multiply by e^x)

$(e^x)^2 + 1 = 4e^x \Rightarrow (e^x)^2 - 4e^x + 1 = 0$

Let $y = e^x \Rightarrow y^2 - 4y + 1 = 0$

* $y = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{4 \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$

$= \dfrac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$

Then $y = 2 + \sqrt{3}$ or $y = 2 - \sqrt{3}$

$e^x = 2 + \sqrt{3}$ or $e^x = 2 - \sqrt{3}$

$\ln e^x = \ln(2 + \sqrt{3})$ or $\ln e^x = \ln(2 - \sqrt{3})$

$x = \ln(2 + \sqrt{3})$ or $x = \ln(2 - \sqrt{3})$

The solutions are $x = \ln(2 + \sqrt{3})$ and $x = \ln(2 - \sqrt{3})$

- بضرب المعادلة بـ e^x
- ثم نعوض عن $y = e^x$ لنتحول الى معادلة تربيعية
- نحل المعادلة التربيعية
- نأخذ كل حل و نساويه بـ e^x
- نأخذ ln للطرفين لايجاد قيمة x

39. $16^x + 4^{x+1} - 3 = 0$

solution:

$(4^2)^x + 4^x \cdot 4 - 3 = 0$

$(4^x)^2 + 4^x \cdot 4 - 3 = 0$

Let $y = 4^x \Rightarrow y^2 + 4y - 3 = 0$

* $y = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-4 \pm \sqrt{(4)^2 - 4(1)(-3)}}{2(1)}$

$= \dfrac{-4 \pm 2\sqrt{7}}{2} = -2 \pm \sqrt{7}$

Then $y = -2 + \sqrt{7}$ or $y = -2 - \sqrt{7}$

$4^x = -2 + \sqrt{7}$ or $e^x = -2 - \sqrt{7}$ has no solution

$\ln 4^x = \ln(-2 + \sqrt{7})$

$x \ln 4 = \ln(-2 + \sqrt{7})$

$x = \dfrac{\ln(-2 + \sqrt{7})}{\ln 4}$

The solution is $x = \dfrac{\ln(-2 + \sqrt{7})}{\ln 4}$

Related Problem (1): Solve the following equation

5. $2\log x - x\log x = 0$

solution:

$$2\log x - x\log x = 0$$
$$(2-x)\log x = 0$$
$$2-x = 0 \quad \text{or} \quad \log x = 0$$
$$x = 2 \quad \text{or} \quad x = 1$$

$x = 1 \in$ Domain of $\log x$

$x = 2 \in$ Domain of $\log x$

The solutions are $x = 1$ and $x = 2$

Related Problem (2): Solve the following equation

5. $x\ln x - \ln(x^2) = 0$

solution:

$$x\ln x - 2\ln(x) = 0$$
$$(x-2)\ln x = 0$$
$$x-2 = 0 \quad \text{or} \quad \ln x = 0$$
$$x = 2 \quad \text{or} \quad x = 1$$

$x = 1 \in$ Domain of $\ln x$

$x = 2 \in$ Domain of $\ln x$

The solutions are $x = 1$ and $x = 2$

Related Problem(5) : Find the x – intercept for the function $f(x) = \log(3x+7) - 1$

solution:

* To find x – intercept ($f(x) = 0$)

$$\log(3x+7) - 1 = 0$$
$$\log(3x+7) = 1$$
$$3x + 7 = 10$$
$$3x = 3$$
$$x = 1$$

The x – intercept is $x = 6$

Exercises 40 – 46 , Find the intercepts of $(f \circ g)(x)$

44. $f(x) = 3^x - 1$, $g(x) = 2x - 5$

solution:

* $(f \circ g)(x) = f(g(x))$

$$= f(2x-5)$$
$$= 3^{2x-5} - 1$$

* To find x – intercept

$$3^{2x-5} - 1 = 0$$
$$3^{2x-5} = 1$$
$$3^{2x-5} = 3^0$$

then $2x - 5 = 0$

$$2x = 5$$
$$x = \frac{5}{2} \in \text{Domain of } (f \circ g)(x)$$

* To find y – intercept

$$(f \circ g)(0) = 3^{2(0)-5} - 1 = -\frac{242}{243}$$

40. $f(x) = 2^{2x-1}$ and $g(x) = 3 + \log_2 x$

solution:

* $(f \circ g)(x) = f(g(x))$

$= f(3 + \log_2 x)$

$= 2^{2(3+\log_2 x)-1}$

$= 2^{6+2\log_2 x-1}$

$= 2^{5+2\log_2 x}$

$= 2^{5+\log_2 x^2}$

$= 2^5 \cdot 2^{\log_2 x^2}$

$= 32x^2$

* To find x – intercept

$32x^2 = 0 \Rightarrow x = 0 \notin$ Domain of $g(x) = 3 + \log_2 x$ and $(f \circ g)(x)$

* To find y – intercept

But $x = 0 \notin$ Domain of $g(x) = 3 + \log_2 x$ and $(f \circ g)(x)$

42. $f(x) = 7^{x-1} - 3$ and $g(x) = \log_7(3 - 8x)$

solution:

* $(f \circ g)(x) = f(g(x))$

$= f(\log_7(3 - 8x))$

$= 7^{\log_7(3-8x)-1} - 3$

$= 7^{\log_7(3-8x)} \cdot 7^{-1} - 3$

$= (3 - 8x) \cdot \frac{1}{7} - 3 = \frac{3-8x}{7} - 3$

* x – intercept

$\frac{3-8x}{7} - 3 = 0$

$\frac{3-8x}{7} = 3$

$3 - 8x = 21$

$-8x = 18 \Rightarrow x = -\frac{9}{4}$

* y – intercept

$(f \circ g)(0) = \frac{3-8(0)}{7} - 3 = -\frac{18}{7}$

Exercises 47 – 50 , Find the inverse for the following functions

47. $f(x) = 2 - e^{x+3}$

solution:

$$y = 2 - e^{x+3}$$
$$2 - e^{x+3} = y$$
$$-e^{x+3} = y - 2 \qquad \text{divide by } (-1)$$
$$e^{x+3} = 2 - y$$
$$\ln e^{x+3} = \ln(2 - y)$$
$$x + 3 = \ln(2 - y)$$
$$x = \ln(2 - y) - 3 \qquad \Rightarrow \quad f^{-1}(y) = \ln(2 - y) - 3$$

Then the inverse is $f^{-1}(x) = \ln(2 - x) - 3$

48. $f(x) = \ln(1 - x) - 4$

solution:

$$\ln(1 - x) - 4 = y$$
$$\ln(1 - x) = y + 4$$
$$1 - x = e^{y+4}$$
$$-x = e^{y+4} - 1$$
$$x = 1 - e^{y+4} \qquad \Rightarrow \quad f^{-1}(y) = 1 - e^{y+4}$$

The inverse is $f^{-1}(x) = 1 - e^{x+4}$

Ex : The growth of a certain bacteria in a culture is given by the model

$$P(t) = 400e^{0.05t}$$

where $P(t)$ is the number of bacteria at time t (in minutes)

a. What is the initial number of bacteria.

b. What is the population after half an hour?

c. How long will it take for the population to reach 1200.

d. What is the doubling time for the population ?

نمو عينة من بكتيريا يعطى حسب العلاقة

$P(t) = 400e^{0.05t}$ حيث p(t) عدد البكتريا عند الزمن t

a- ما العدد الابتدائي للبكتيريا

b- ما العدد بعد نصف ساعة

c- ما المدة اللازمة لوصول العدد 1200

d- ما الوقت لمضاعفة عدد العينة

solution:

a. The initial number when $t = 0$

$P(0) = 400e^{0.05(0)} = 400$

b. half an hour $= 30$ minutes

$P(30) = 400e^{0.05(30)} = 1793$

c. $P(t) = 400e^{0.05t}$, find t when $P = 1200$

$1200 = 400e^{0.05t}$ divide by 400

$3 = e^{0.05t}$ take ln of sides

$\ln 3 = \ln e^{0.05t}$

$\ln 3 = 0.05t$

$t = \dfrac{\ln 3}{0.05} = 21.97$ minutes

d. $P(t) = 400e^{0.05t}$, find t when the population doubled $P = 800$

$800 = 400e^{0.05t}$ divide by 400

$2 = e^{0.05t}$ take ln of sides

$\ln 2 = \ln e^{0.05t}$

$\ln 2 = 0.05t$

$t = \dfrac{\ln 2}{0.05} = 13.86$ minutes

Example (4): A population grows from 100 to 130 in 2 weeks . Find the growth rate؟

solution:

The model $P(t) = ae^{rt}$

We have $P(0) = 100$ and $P(2) = 130$

* $P(0) = 100 \Rightarrow P(t) = ae^{rt}$

$100 = ae^{0} \rightarrow a = 100$

* $P(2) = 130 \Rightarrow P(t) = 100e^{rt}$

$130 = 100e^{2r}$ divide by 100

$1.3 = e^{2r}$

$\ln 1.3 = \ln e^{2r}$

$\ln 1.3 = 2r$ divide by (2)

$0.1312 = r$

The population is growing at a rate ($0.1312 \cdot 100 = 13.12\%$ per week)

CHPTER 5 : TRIGONOMETRIC FUNCTIONS

Section (5 – 1) : ANGLES AND RADIAN MEASURE

Exercise 1 – 2 , Convert the degree measures to radians

1.

للتحويل من درجات الى راديان نضرب الدرجة بـ $\frac{\pi}{180}$

a. 150°

solution:

$* \ 150^\circ = 150 \times \frac{\pi}{180} = \frac{5\pi}{6}$ radian

b. 120°

solution:

$* \ 120^\circ = 120 \times \frac{\pi}{180} = \frac{2\pi}{3}$ radian

c. 450°

solution:

$* \ 450^\circ = 450 \times \frac{\pi}{180} = \frac{5\pi}{2}$ radian

d. –135°

solution:

$* \ -135^\circ = -135 \times \frac{\pi}{180} = -\frac{3\pi}{4}$ radian

e. 630°

solution:

$* \ 630^\circ = 630 \times \frac{\pi}{180} = \frac{7\pi}{2}$ radian

Exercise 3 – 4 , Convert the radian measures to degree

للتحويل من راديان الى درجات نضرب الدرجة بـ $\frac{\pi}{180}$

a. $\frac{2\pi}{3}$

solution:

* $\frac{2\pi}{3} = \frac{2\pi}{3} \times \frac{180}{\pi} = 120^{\circ}$

b. $\frac{5\pi}{6}$

solution:

* $\frac{5\pi}{6} = \frac{5\pi}{6} \times \frac{180}{\pi} = 150^{\circ}$

c. $-\frac{3\pi}{4}$

solution:

* $-\frac{3\pi}{4} = -\frac{3\pi}{4} \times \frac{180}{\pi} = -135^{\circ}$

d. $\frac{7\pi}{2}$

solution:

* $\frac{7\pi}{2} = \frac{7\pi}{2} \times \frac{180}{\pi} = 630^{\circ}$

e. $\frac{7\pi}{3}$

solution:

* $\frac{7\pi}{3} = \frac{7\pi}{3} \times \frac{180}{\pi} = 420^{\circ}$

5. Through how many complete revolation does a bicycle with raduis one decimeter turn when the bicycle travels 440 meter.

solution

كم عدد الدورات الكاملة لدراجة نصف قطرها 1 ديسمتر عندما تحركت الدراجة 440 متر

1 ديسمتر = $\frac{1}{10}$ متر

نقسم المسافة (طول المنحنى S) على نصف القطر لنحصل على زاوية الدوران ثم نقسم زاوية الدوران على 2π لنحصل على عدد الدورات

$$\theta = \frac{S}{r} = \frac{440}{0.1} = 4400$$

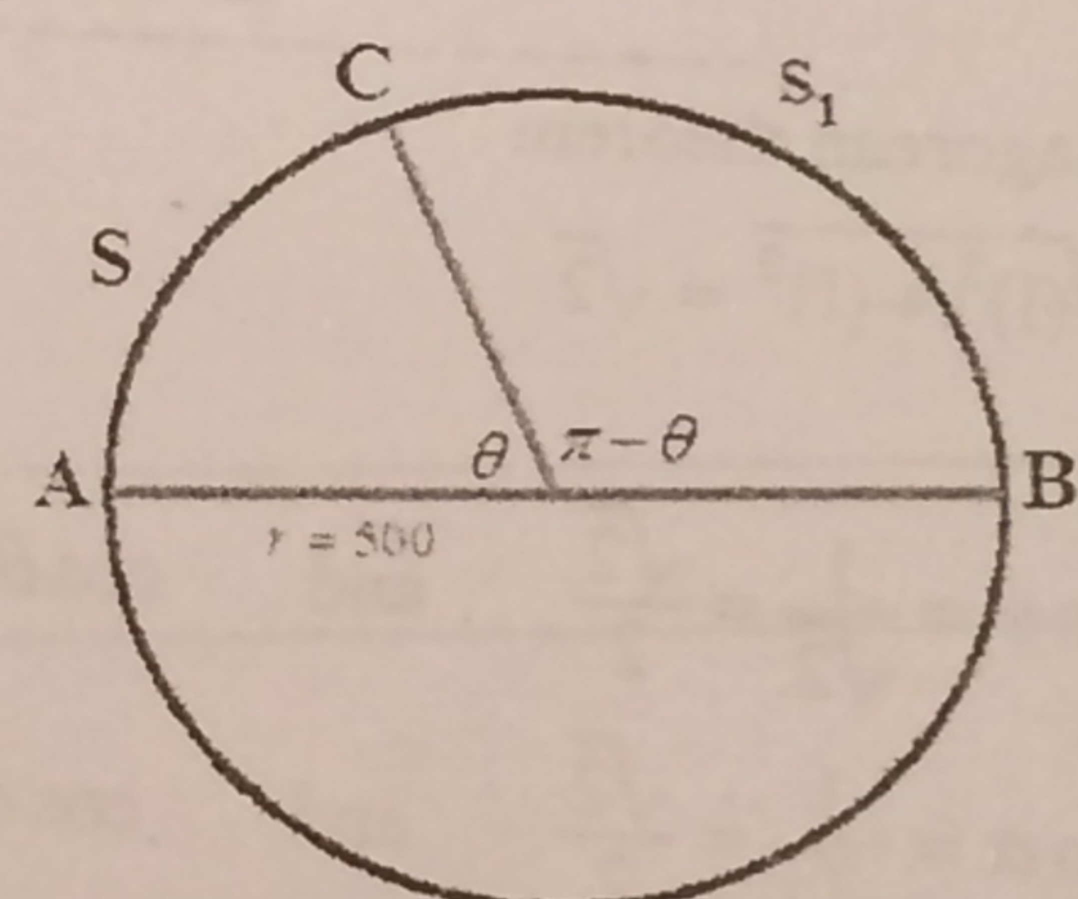
* Number of revolutions $= \frac{4400}{2\pi} = 700$ revolutions

6. Consider a racetrack with radius 500 decimeter . Suppose a straight path connects two points A and B , diametrically opposite one another , on the track . If a man is 660 decimeter around the track from point A . How far from the path is the man

اعتبر مضمار سباق نصف قطره 500 ديسمتر . لنفرض مسار مستقيم يصل بالنقطتين A و B متعاكسين تماما (قطر) . اذا دار رجل 660 ديسمتر حول المضمار من النقطة A . ما البعد بين الرجل و المسار

solution:

$$\theta = \frac{S}{r} = \frac{660}{500} = 1.32 \text{ radian}$$



* $S_1 = \overset{\frown}{CB} = (\pi-\theta)\cdot r$

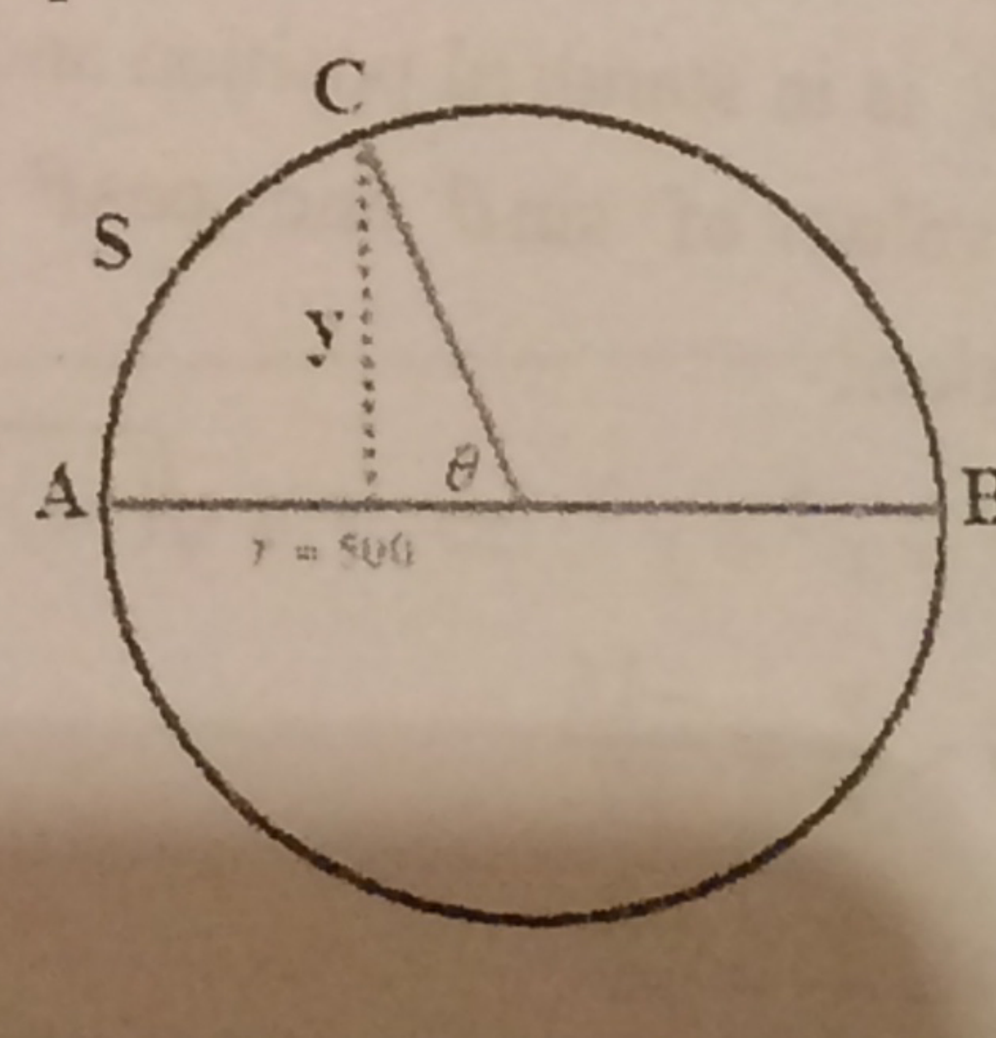
$= (\pi - 1.32)\cdot 500$

$= 910.8$ decimeter

6. Consider a racetrack with radius 500 decimeter . Suppose a straight path connects two points A and B , diametrically opposite one another , on the track . If a man is 660 decimeter around the track from point A . How far from the path is the man

solution:

$$\theta = \frac{S}{r} = \frac{660}{500} = 1.32 \text{ radian}$$



* $y = r\sin\theta$

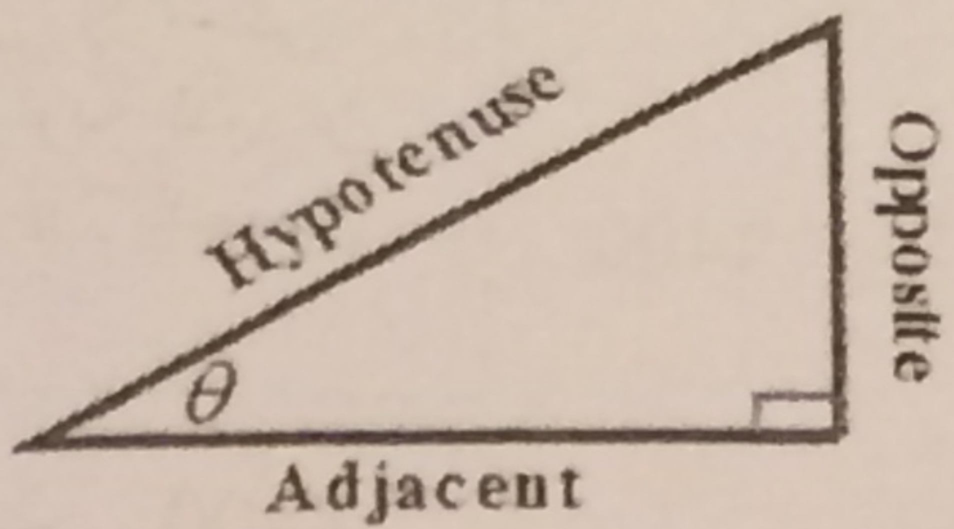
$= 500\ \sin(1.32)$

$= 484.36$ decimeter

راجع المحاضر : أي الحلين هو المطلوب

DEFINITION : Sine and Cosine Functions for a cute angle

* $\sin\theta = \dfrac{\text{Opposite}}{\text{Hypotenuse}}$
* $\cos\theta = \dfrac{\text{adjacent}}{\text{Hypotenuse}}$

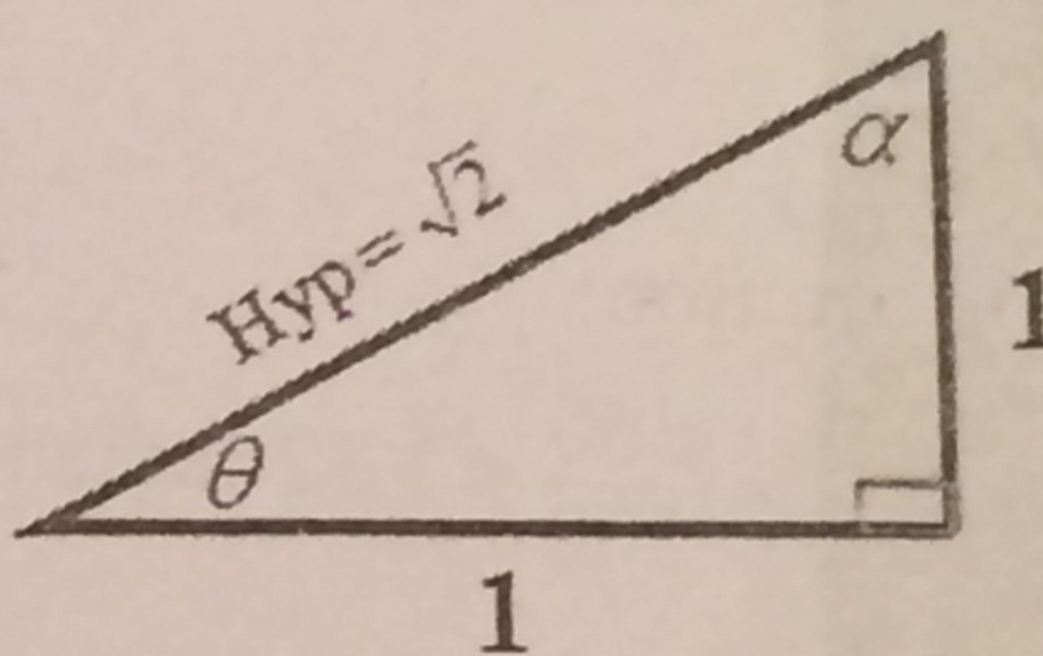


RELATED PROBLEM (1)

A right triangle whose opposite and adjacent sides are both have measure 1 unit. Find the triangle angles in radian measures and calculate both sine and cosine of those angles

solution:

نرسم المثلث القائم و نضع كل من المقابل و
المجاور 1 ثم نوجد الوتر بنظرية فيثاغورس . ثم
نبحث بالجدول بالصفحة التالية عن الزوايا التي
تحقق قيم كل من sin and cos



* By the Pythagorean theorem

$$\text{Hyp} = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$$

We have $\sin\theta = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$ and $\cos\theta = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$ $\Rightarrow \theta = \dfrac{\pi}{4}$

We have $\sin\alpha = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$ and $\cos\alpha = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$ $\Rightarrow \alpha = \dfrac{\pi}{4}$

* $\sin\left(\dfrac{\pi}{4}\right) = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$, $\cos\left(\dfrac{\pi}{4}\right) = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

Related Problem (2)

If θ is in standard position and $Q(-8,-15)$ is on the terminal side of θ . Find the values of $\sin\theta$ and $\cos\theta$

solution:

$$r^2 = x^2 + y^2 \Rightarrow r = \sqrt{(-8)^2 + (-15)^2} = 17$$

* $\sin\theta = \dfrac{y}{r} = \dfrac{-15}{17}$
* $\cos\theta = \dfrac{x}{r} = \dfrac{-8}{17}$

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	0	$\frac{1}{2}$	1

Periodic Function الدالة الدورية

1- Periodic Functions with period 2π

* $\sin(x) = \sin(x + 2n\pi)$
* $\cos(x) = \cos(x + 2n\pi)$
* $\sec(x) = \sec(x + 2n\pi)$
* $\csc(x) = \csc(x + 2n\pi)$

2- Periodic Functions with period π

* $\tan(x) = \tan(x + n\pi)$
* $\cot(x) = \cot(x + n\pi)$

Other Trigonometric Functions

1. $\tan x = \dfrac{\sin x}{\cos x}$
2. $\cot x = \dfrac{\cos x}{\sin x}$, $\cot x = \dfrac{1}{\tan x}$
3. $\sec x = \dfrac{1}{\cos x}$
4. $\csc x = \dfrac{1}{\sin x}$

0559108708

Exercises 1 – 5 , Determine exact function value

1.

a. $\sin\left(\frac{\pi}{6}\right)$

solution:

$\sin\left(\frac{\pi}{6}\right)=\frac{1}{2}$

b. $\cos\left(\frac{\pi}{4}\right)$

solution:

$\cos\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$

c. $\sin\left(\frac{-3\pi}{4}\right)$

$2-\frac{3}{4}=\frac{5}{4}$

$-\frac{3}{4}=\frac{5}{4}-2$

$-\frac{3\pi}{4}=\frac{5\pi}{4}-2\pi$

solution:

$$\sin\left(\frac{-3\pi}{4}\right)=\sin\left(\frac{5\pi}{4}+(-1)(2\pi)\right)$$

$$=\sin\left(\frac{5\pi}{4}\right)$$

لإيجاد القيمة نستخدم الجدول

$$=-\frac{\sqrt{2}}{2}$$

d. $\cos\left(-\frac{\pi}{3}\right)$

$2-\frac{1}{3}=\frac{5}{3}$

$-\frac{1}{3}=\frac{5}{3}-2$

$-\frac{\pi}{3}=\frac{5\pi}{3}-2\pi$

solution:

$$* \cos\left(-\frac{\pi}{3}\right)=\cos\left(\frac{5\pi}{3}+(-1)(2\pi)\right)$$

$$=\cos\left(\frac{5\pi}{3}\right)$$

$$=\frac{1}{2}$$

ملحوظة هامة جدا :
يتم حل هذه الامثلة بالسكشن التالي بقوانين أخرى و المفروض تحل بها بالاختبار . انظر المراجعة النهائية بآخر المذكرة

0559108708 محمد ندا (ابو يوسف)

2.

a. $\cos\left(\frac{\pi}{3}\right)$

solution:

$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

b. $\sin\left(\frac{\pi}{4}\right)$

solution:

$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

c. $\sin(-2\pi)$

solution:

$$\begin{aligned} * \sin(-2\pi) &= \sin(0 + (-1)(2\pi)) \\ &= \sin(0) \\ &= 0 \end{aligned}$$

d. $\cos\left(-\frac{\pi}{2}\right)$

solution:

$$\begin{aligned} * \cos\left(-\frac{\pi}{2}\right) &= \cos\left(\frac{3\pi}{2} + (-1)(2\pi)\right) \\ &= \cos\left(\frac{3\pi}{2}\right) \\ &= -1 \end{aligned}$$

$2 - \frac{1}{2} = \frac{3}{2}$

$-\frac{1}{2} = \frac{3}{2} - 2$

$-\frac{\pi}{2} = \frac{3\pi}{2} - 2\pi$

4.

a. $\sec(\pi)$

solution:

$$* \ \sec(\pi) = \frac{1}{\cos(\pi)}$$

$$= \frac{1}{-1} = -1$$

b. $\cot\left(\frac{\pi}{6}\right)$

solution:

$$* \ \cot\left(\frac{\pi}{6}\right) = \frac{\cos\left(\frac{\pi}{6}\right)}{\sin\left(\frac{\pi}{6}\right)}$$

$$= \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

c. $\csc\left(-\frac{2\pi}{3}\right)$

solution:

$$* \ \csc\left(-\frac{2\pi}{3}\right) = \csc\left(\frac{4\pi}{3} + (-1)(2\pi)\right)$$

$$= \csc\left(\frac{4\pi}{3}\right)$$

$$= \frac{1}{\sin\left(\frac{4\pi}{3}\right)}$$

$$= \frac{1}{-\frac{\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}}$$

$$2 - \frac{2}{3} = \frac{4}{3}$$

$$-\frac{2}{3} = \frac{4}{3} - 2$$

$$-\frac{2\pi}{3} = \frac{4\pi}{3} - 2\pi$$

d. $\tan\left(-\frac{\pi}{4}\right)$

حل هذا المثال بالسكشن القادم بيكون أسهل

$$-\frac{1}{4} = \frac{3}{4} - 1$$

$$-\frac{\pi}{4} = \frac{3\pi}{4} - \pi$$

solution:

$$* \ \tan\left(-\frac{\pi}{4}\right) = \tan\left(\frac{3\pi}{4} + (-1)(\pi)\right)$$

لاحظ أن دورة tan هي π

$$= \tan\left(\frac{3\pi}{4}\right)$$

$$= \frac{\sin\frac{3\pi}{4}}{\cos\frac{3\pi}{4}}$$

$$= \frac{1}{-1} = -1$$

Exercises 6 – 8 , Use the periodicity of sine , cosine , secant , tangent , cotangent , and cosecant as well as their values when $0 \le x \le \pi$ to find the exact value of each of the following .

6.

a. $\sin(8\pi)$

solution:

* $\sin(8\pi) = \sin(0 + 4(2\pi))$

$= \sin(0)$

$= 0$

b. $\cos(10\pi)$

solution:

* $\cos(10\pi) = \cos(0 + 5(2\pi))$

$= \cos(0)$

$= 1$

c. $\sin\left(\frac{17\pi}{2}\right)$

solution:

* $\sin\left(\frac{17\pi}{2}\right) = \sin\left(\frac{\pi}{2} + \frac{16\pi}{2}\right)$

$= \sin\left(\frac{\pi}{2} + 4(2\pi)\right)$

$= \sin\left(\frac{\pi}{2}\right)$

$= 1$

$\frac{17}{2} = \frac{1}{2} + \frac{16}{2}$

$\frac{17\pi}{2} = \frac{\pi}{2} + \frac{16\pi}{2}$

d. $\csc(9\pi)$

solution:

* $\csc(9\pi) = \csc(\pi + 8\pi)$

$9\pi = \pi + 8\pi$

$= \csc(\pi + 2(4\pi))$

$= \csc(\pi)$

$= \frac{1}{\sin \pi}$

$= \frac{1}{0}$ undefined

8.

a. $\sin\left(-\frac{7\pi}{2}\right)$

solution:

$$* \quad \sin\left(-\frac{7\pi}{2}\right) = \sin\left(\frac{\pi}{2} + (-1)4\pi\right)$$

$$= \sin\left(\frac{\pi}{2}\right) = 1$$

b. $\cot\left(-\frac{5\pi}{6}\right)$

$\cot\left(\frac{11\pi}{6} + (-1)\pi\right.$

solution:

لاحظ أن دورة cot هي π

$= \cot\left(\frac{\pi}{3}\right)$

$= \sqrt{3}$

$$* \quad \cot\left(-\frac{5\pi}{6}\right) = \cot\left(\frac{\pi}{6} + (-1)\pi\right)$$

$$= \cot\left(\frac{\pi}{6}\right)$$

$$= \frac{\cos\left(\frac{\pi}{6}\right)}{\sin\left(\frac{\pi}{6}\right)} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

c. $\tan(8\pi)$

solution:

$$* \quad \tan(8\pi) = \tan(0 + 8(\pi))$$

$$= \tan(0)$$

$$= \frac{\sin(0)}{\cos(0)}$$

$$= \frac{0}{1} = 0$$

$= 0$

d. $\cot\left(\frac{7\pi}{4}\right)$

solution:

$$* \quad \cot\left(\frac{7\pi}{4}\right) = \cot\left(\frac{3\pi}{4} + \pi\right)$$

لاحظ أن دورة cot هي π

$$= \cot\left(\frac{3\pi}{4}\right)$$

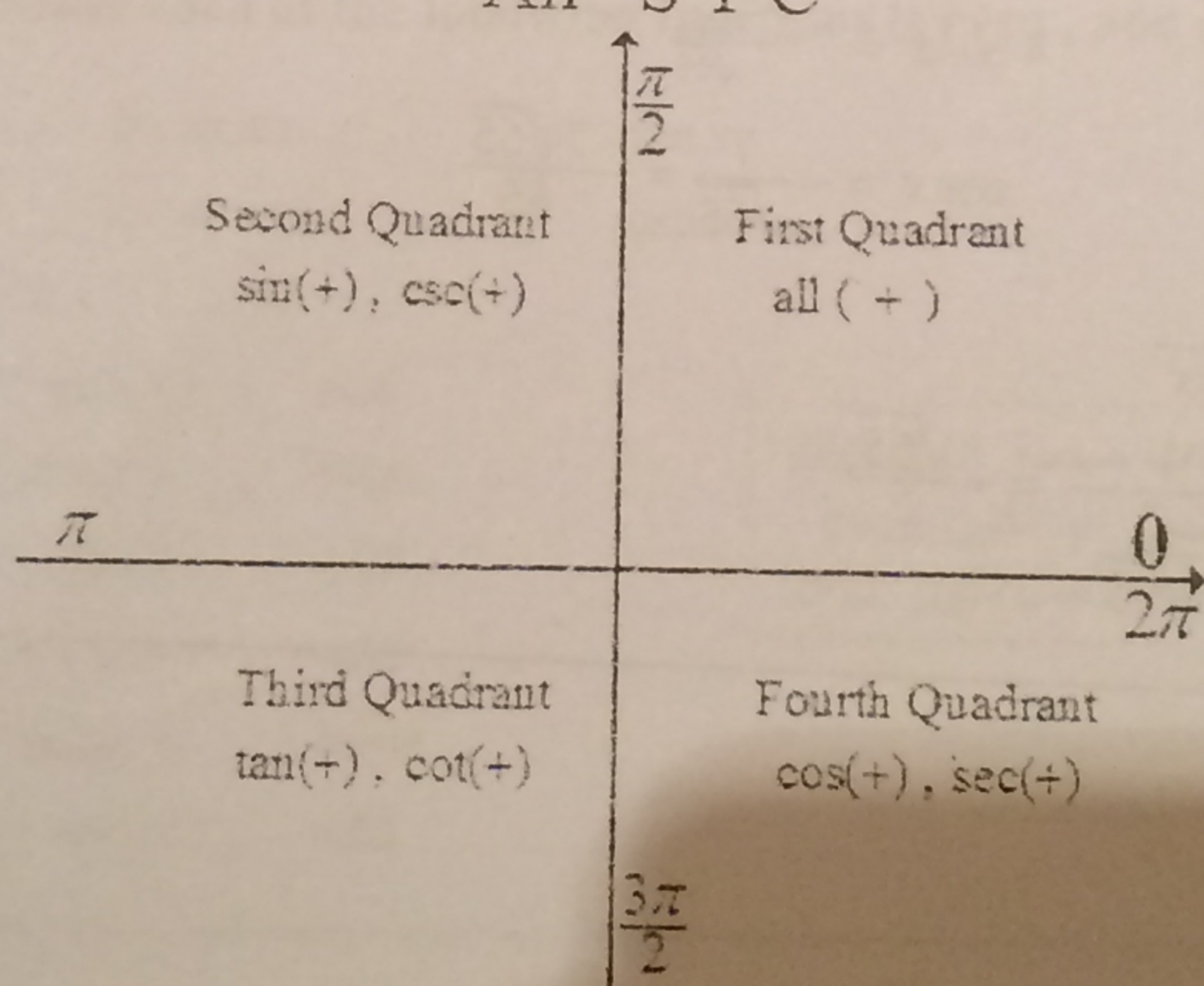
$$= \frac{\cos\left(\frac{3\pi}{4}\right)}{\sin\left(\frac{3\pi}{4}\right)}$$

$$= \frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = -1$$

Exercise 9 ; For each of the following intervals , state which of the six trigonometric functions have positive values through the interval .

a. $\left(0, \frac{\pi}{2}\right)$ solution: sine(+) , cosine (+) , tangent (+) , secant (+) , cosecant(+) , cotangent (+)
b. $\left(\frac{\pi}{2}, \pi\right)$ solution: sine(+) , cosine (−) , tangent (−) , secant (−) , cosecant(+) , cotangent (−)
c. $\left(\pi, \frac{3\pi}{2}\right)$ solution: sine(−) , cosine (−) , tangent (+) , secant (−) , cosecant(−) , cotangent (+)
d. $\left(\frac{3\pi}{2}, 2\pi\right)$ solution: sine(−) , cosine (+) , tangent (−) , secant (+) , cosecant(−) , cotangent (−)

All STC



Exercises 10 – 13 , Find the values of the remaining trigonometric functions .
Under the given condition

10. $\sin x = \frac{4}{5}$ and $\cos x = -\frac{3}{5}$

solution:

$$* \ \tan x = \frac{\sin x}{\cos x}$$

$$= \frac{\frac{4}{5}}{-\frac{3}{5}} = -\frac{4}{3}$$

$$* \ \csc x = \frac{1}{\sin x} = \frac{5}{4}$$

$$* \ \sec x = \frac{1}{\cos x} = -\frac{5}{3}$$

$$* \ \cot x = \frac{1}{\tan x} = -\frac{3}{4}$$

13. $\csc x = -\frac{1}{4}\sqrt{65}$ and $\cot x = \frac{7}{4}$

solution:

$$\csc x = -\frac{1}{4}\sqrt{65} \Rightarrow \sin x = \frac{1}{\csc x} = -\frac{4}{\sqrt{65}}$$

$$\cot x = \frac{7}{4} \Rightarrow \tan x = \frac{1}{\cot x} = \frac{4}{7}$$

$$* \ \tan x = \frac{\sin x}{\cos x} \Rightarrow \frac{4}{7} = \frac{-\frac{4}{\sqrt{65}}}{\cos x}$$

نعوض ثم نضرب طرفين و وسطين

$$4\cos x = 7 \cdot -\frac{4}{\sqrt{65}}$$

$$\cos x = -\frac{7}{\sqrt{65}} = -\frac{7\sqrt{65}}{65}$$

$$* \ \sec x = \frac{1}{\cos x}$$

$$= \frac{1}{-\frac{7}{\sqrt{65}}} = -\frac{\sqrt{65}}{7}$$

Exercises 14 – 18 , Solve the equation for x in $\left[0,2\pi\right]$

17. $\cos 2x = \cos x$

نبحث بالجدول عن زاويتين لهما نفس قيمة cos أحدهما ضعف الأخرى

solution:

* We have $\cos(0)=1$ and $\cos(2(0))=1$

$\cos(0)=\cos(2(0)) \Rightarrow x=0$ is a solution

* We have $\cos\left(\frac{2\pi}{3}\right)=-\frac{1}{2}$ and $\cos\left(\frac{4\pi}{3}\right)=-\frac{1}{2}$

$\cos\left(\frac{2\pi}{3}\right)=\cos\left(2\left(\frac{2\pi}{3}\right)\right) \Rightarrow x=\frac{2\pi}{3}$ is a solution

The solutions are $x=0$ and $x=\frac{2\pi}{3} \in [0,\pi]$

18. $\sec x = -\frac{2\sqrt{3}}{3}$

solution:

$\sec x = -\frac{2\sqrt{3}}{3}$ then $\cos x = -\frac{3}{2\sqrt{3}} = -\frac{\sqrt{3}}{2}$

نبحث بالجدول عن الزوايا التي cos لها $-\frac{\sqrt{3}}{2}$

we have $\cos\left(\frac{5\pi}{6}\right)=-\frac{\sqrt{3}}{2}$ and $\cos\left(\frac{7\pi}{6}\right)=-\frac{\sqrt{3}}{2}$

The solutions are $x=\frac{5\pi}{6}$, $x=\frac{7\pi}{6} \in [0,\pi]$

19. Decide whether each of the following functions is even , odd or neither .

a. $\sin x$ b. $\cos x$ c. $\tan x$

d. $\cot x$ e. $\sec x$ f. $\csc x$

solution:

a. $\sin(-x)=-\sin(x)$, odd

b. $\cos(-x)=\cos(x)$, even

c. $\tan(-x)=-\tan(x)$, odd

d. $\cot(-x)=-\cot(x)$, odd

e. $\sec(-x)=\sec(x)$, even

f. $\csc(-x)=-\csc(x)$, odd

انظر تمثيل الدوال بالكتاب صفحة 248, 246
الدوال المتماثلة حول محور y تكون even
الدوال المتماثلة حول نقطة الأصل تكون odd

Exercises 20 – 23 , Solve the inequality for x in $[0, 2\pi]$

20. $\sin x \geq 0$

solution

$x \in [0, \pi]$

نبحث عن الفترة التي تكون فيها قيمة sin موجبة و ذلك بالربع الأول و الثاني

23. $\sec x \leq 0$

solution:

$\sec x \leq 0 \Rightarrow \cos x \leq 0$

$x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

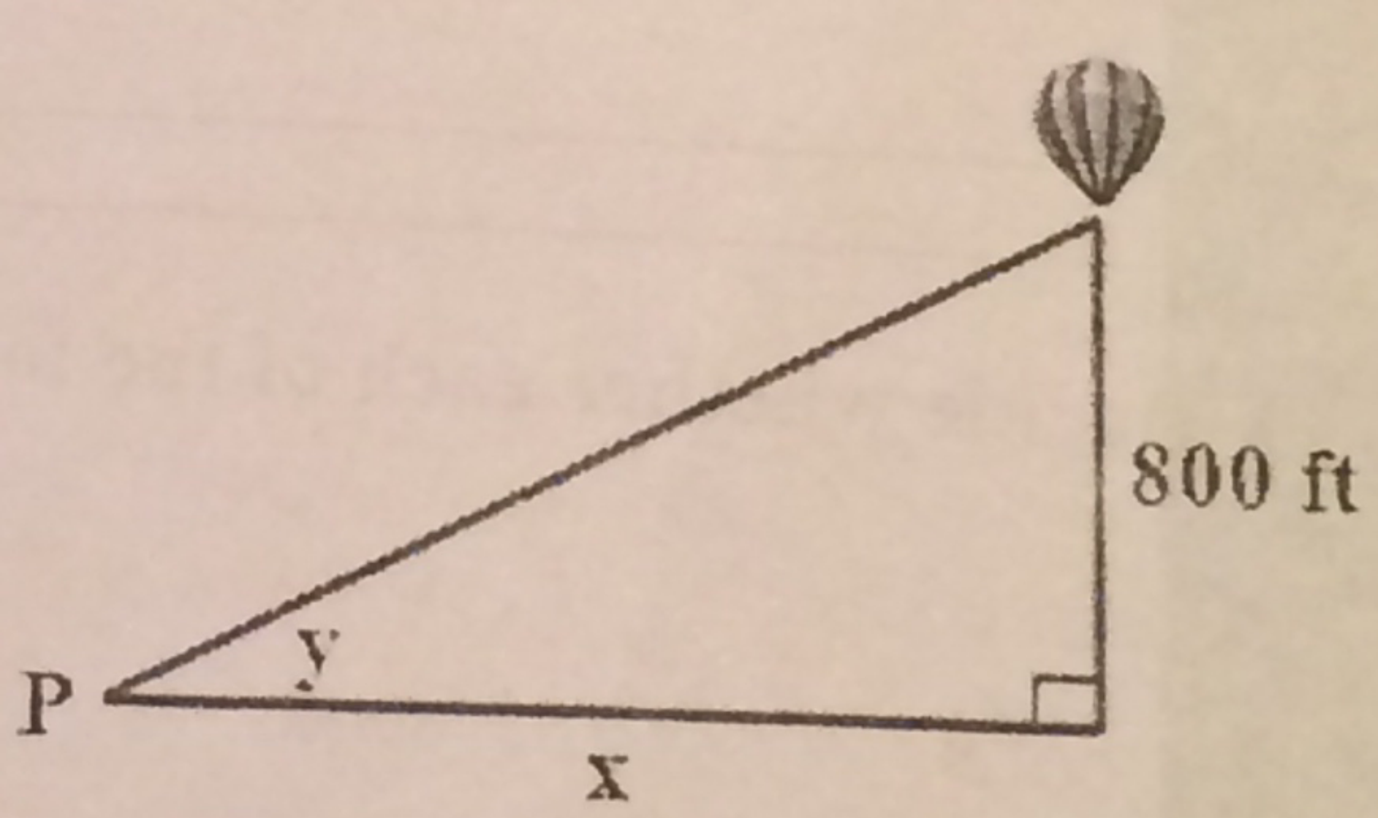
بما أن sec سالبة فان cos تكون سالب
نبحث عن الفترة التي تكون فيها قيمة cos سالبة
و ذلك بالربع الثاني و الثالث

24. A hot - air balloon over Albuquerque , New Mexico , is being blown due east from point P and traveling at a constant height of 800 ft , The angle y is formed by the ground and the line of vision from P to the balloon . This angle changes as the balloon travels see the figure

a. Express the horizontal distance x as a function of the angle y.

b. When the angle is 20 radians , what is its horizontal distance from P?

c. An angle of 20 radians is equivalent to how many degrees.



solution:

a. $\tan y = \dfrac{800}{x} \Rightarrow x = \dfrac{800}{\tan y}$

b. $x = \dfrac{800}{\tan(20)} = 357.6$ ft

c. 20 radians $= 20 - 3(2\pi) = 1.15$ radians

* 1.15 radians $= 1.15 \cdot \dfrac{180}{\pi} = 65$

Exercise 25. If θ is in standard position and $Q\left(\frac{3}{5},\frac{4}{5}\right)$ is on the terminal side of θ. Use Definition 5.2.2 to find the values of $\sin\theta$ and $\cos\theta$

solution:

$$x^2+y^2=\left(\frac{3}{5}\right)^2+\left(\frac{4}{5}\right)^2=1$$

* $\sin\theta=y=\frac{4}{5}$

* $\cos\theta=x=\frac{3}{5}$

Basic Identities

1. $\sin^2 x + \cos^2 x = 1$

2. $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$

3. $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

4. $\sin(-x) = -\sin(x)$

5. $\cos(-x) = \cos(x)$

6. $\tan^2 x + 1 = \sec^2 x$

7. $1 + \cot^2 x = \csc^2 x$

8. $\sin(x + 2\pi) = \sin(x)$

9. $\cos(x + 2\pi) = \cos(x)$

10. $\sin\left(\frac{\pi}{2} - x\right) = \cos(x)$ and $\sin\left(\frac{\pi}{2} + x\right) = \cos(x)$

11. $\cos\left(\frac{\pi}{2} - x\right) = \sin(x)$ and $\cos\left(\frac{\pi}{2} + x\right) = -\sin(x)$

12. $\sin(2x) = 2\sin x \cos x$

13. $\cos^2 x = \frac{1}{2}(1 + \cos(2x))$

14. $\sin^2 x = \frac{1}{2}(1 - \cos(2x))$

15. $\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$

Exercise 1. Using identities , Prove that the following identities hold:

a. $\sin(\pi - x) = \sin x$

solution:

* $\sin(\pi - x) = \sin\pi\cos x - \cos\pi\sin x$

$\boxed{\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y}$

$= (0)\cos x - (-1)\sin x$

$= \sin x$

b. $\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$

solution:

* $\sin\left(\frac{3\pi}{2} - x\right) = \sin\left(\frac{3\pi}{2}\right)\cos x - \cos\left(\frac{3\pi}{2}\right)\sin x$

$\boxed{\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y}$

$= (-1)\cos x - (0)\sin x$

$= -\cos x$

c. $\cos(\pi - x) = -\cos x$

solution:

* $\cos(\pi - x) = \cos(\pi)\cos(x) + \sin(\pi)\sin(x)$

$\boxed{\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y}$

$= (-1)\cos x + (0)\sin(x)$

$= -\cos x$

d. $\tan\left(\frac{3\pi}{2} - x\right) = \cot x$

solution:

* $\tan\left(\frac{3\pi}{2} - x\right) = \dfrac{\sin\left(\frac{3\pi}{2} - x\right)}{\cos\left(\frac{3\pi}{2} - x\right)}$

$= \dfrac{\sin\frac{3\pi}{2}\cos x - \cos\frac{3\pi}{2}\sin x}{\cos\frac{3\pi}{2}\cos x + \sin\frac{3\pi}{2}\sin x}$

$= \dfrac{(-1)\cos x - (0)\sin x}{(0)\cos x + (-1)\sin x}$

$= \dfrac{-\cos x}{-\sin x}$

$= \cot x$

$e.\ \cos\left(\frac{3\pi}{2}-x\right)=-\sin x$

solution:

$$*\ \cos\left(\frac{3\pi}{2}-x\right)=\cos\tfrac{3\pi}{2}\cos x+\sin\tfrac{3\pi}{2}\sin x$$

$$=(0)\cos x+(-1)\sin x$$

$$=-\sin x$$

$f.\ \tan(\pi-x)=-\tan x$

solution:

$$*\ \tan(\pi-x)=\frac{\sin(\pi-x)}{\cos(\pi-x)}$$

$$=\frac{\sin\pi\cos x-\cos\pi\sin x}{\cos\pi\cos x+\sin\pi\sin x}$$

$$=\frac{(0)\cos x-(-1)\sin x}{(-1)\cos x+(0)\sin x}$$

$$=\frac{\sin x}{-\cos x}$$

$$=-\tan x$$

3. Using identity (1) , show that $\cos x=\sqrt{1-\sin^2 x}$ for $0\le x\le\frac{\pi}{2}$,

Show also that $\cos x=-\sqrt{1-\sin^2 x}$ for $\frac{\pi}{2}\le x\le\frac{3\pi}{2}$

solution:

اشارة cos بالربع الأول موجبة
اشارة cos بالربع الثاني والثالث سالبة

$$\sin^2 x+\cos^2 x=1$$

$$\cos^2 x=1-\sin^2 x$$

$$\cos x=\pm\sqrt{1-\sin^2 x}$$

* $\cos x$ is (+) for $0\le x\le\frac{\pi}{2}$ $\Rightarrow$ $\cos x=\sqrt{1-\sin^2 x}$ for $0\le x\le\frac{\pi}{2}$

* $\cos x$ is (−) for $\frac{\pi}{2}\le x\le\frac{3\pi}{2}$ $\Rightarrow$ $\cos x=-\sqrt{1-\sin^2 x}$ for $\frac{\pi}{2}\le x\le\frac{3\pi}{2}$

0559108708 (محمد ندا (ابو يوسف

4. Using identity (1), show that $\sin x = \sqrt{1-\cos^2 x}$ for $0 \le x \le \pi$,

Show also that $\sin x = -\sqrt{1-\cos^2 x}$ for $\pi \le x \le 2\pi$

solution:

اشارة sin بالربع الأول و الثاني موجبة
اشارة sin بالربع الثالث و الرابع سالبة

$$\sin^2 x + \cos^2 x = 1$$
$$\sin^2 x = 1 - \cos^2 x$$
$$\sin x = \pm\sqrt{1-\cos^2 x}$$

* $\sin x$ is (+) for $0 \le x \le \pi \Rightarrow \sin x = \sqrt{1-\cos^2 x}$ for $0 \le x \le \pi$
* $\sin x$ is (−) for $\pi \le x \le 2\pi \Rightarrow \sin x = -\sqrt{1-\cos^2 x}$ for $\pi \le x \le 2\pi$

Exercises 5 – 8 , Find the exact value

5.

a. $\sin\left(\frac{2\pi}{3}\right)$

نقسم الزاوية الى جزئيين ($\frac{\pi}{2}$ + زاوية)
ثم نستخدم القاعدة $\sin\left(\frac{\pi}{2}+x\right) = \cos(x)$

* $\frac{2}{3} - \frac{1}{2} = \frac{1}{6}$

solution:

$$* \ \sin\left(\frac{2\pi}{3}\right) = \sin\left(\frac{\pi}{2}+\frac{\pi}{6}\right)$$
$$= \cos\left(\frac{\pi}{6}\right)$$
$$= \frac{\sqrt{3}}{2}$$

b. $\sin\left(-\frac{5\pi}{4}\right)$

solution:

نستخدم القاعدة $\sin(-x) = -\sin(x)$

$$* \ \sin\left(-\frac{5\pi}{4}\right) = -\sin\left(\frac{5\pi}{4}\right)$$
$$= -\sin\left(\pi + \frac{\pi}{4}\right)$$
$$= -\left[\sin(\pi)\cos(\tfrac{\pi}{4}) + \cos(\pi)\sin(\tfrac{\pi}{4})\right]$$
$$= -\left[(0)\left(\tfrac{\sqrt{2}}{2}\right) + (-1)\left(\tfrac{\sqrt{2}}{2}\right)\right]$$
$$= +\frac{\sqrt{2}}{2}$$

اذا كانت الزاوية بالربع الثالث نقسم الزاوية الى (π + زاوية)
ثم نستخدم القاعدة
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$

ملحوظة هامة : يتم حل التمارين حسب المتطابقات الموجودة بالخطة الموجودة بالكتاب صفحة 252 و 253 . لأن يمكن حل التمارين بمتطابقات أسهل لكنها غير موجودة بالمتطابقات بالكتاب و لذلك التزمت بطرق الكتاب

7.

a. $\tan\left(\frac{5\pi}{6}\right)$

solution:

$$* \quad \tan\left(\frac{5\pi}{6}\right)=\frac{\sin\left(\frac{5\pi}{6}\right)}{\cos\left(\frac{5\pi}{6}\right)}$$

$$=\frac{\sin\left(\frac{\pi}{2}+\frac{\pi}{3}\right)}{\cos\left(\frac{\pi}{2}+\frac{\pi}{3}\right)}$$

$$=\frac{\cos\left(\frac{\pi}{3}\right)}{-\sin\left(\frac{\pi}{3}\right)}$$

$$=-\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}=-\frac{1}{\sqrt{3}}$$

$$\boxed{\begin{aligned}\sin\left(\frac{\pi}{2}+x\right)&=\cos(x)\\ \cos\left(\frac{\pi}{2}+x\right)&=-\sin(x)\end{aligned}}$$

b. $\tan\left(-\frac{\pi}{3}\right)$

solution:

$$* \quad \tan\left(-\frac{\pi}{3}\right)=\frac{\sin\left(-\frac{\pi}{3}\right)}{\cos\left(-\frac{\pi}{3}\right)}$$

$$=\frac{-\sin\left(\frac{\pi}{3}\right)}{\cos\left(\frac{\pi}{3}\right)}$$

$$=\frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}=-\sqrt{3}$$

$$\boxed{\begin{aligned}&* \ \sin(-x)=-\sin(x)\\ &* \ \cos(-x)=\cos(x)\end{aligned}}$$

8.

a. $\sec\left(\frac{2\pi}{3}\right)$

solution

$$* \quad \sec\left(\frac{2\pi}{3}\right)=\frac{1}{\cos\left(\frac{2\pi}{3}\right)}$$

$$\boxed{* \ \sec x=\frac{1}{\cos x}}$$

$$=\frac{1}{\cos\left(\frac{\pi}{2}+\frac{\pi}{6}\right)}$$

$$\boxed{\cos\left(\frac{\pi}{2}+x\right)=-\sin(x)}$$

$$=\frac{1}{-\sin\left(\frac{\pi}{6}\right)}$$

$$=\frac{1}{-\frac{1}{2}}=-2$$

b. $\sec\left(-\frac{\pi}{6}\right)$

solution:

* $\sec\left(-\frac{\pi}{6}\right) = \frac{1}{\cos\left(-\frac{\pi}{6}\right)}$

$= \frac{1}{\cos\left(\frac{\pi}{6}\right)}$ $\quad \boxed{\cos(-x) = \cos(x)}$

$= \frac{1}{\frac{\sqrt{3}}{2}}$

$= \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

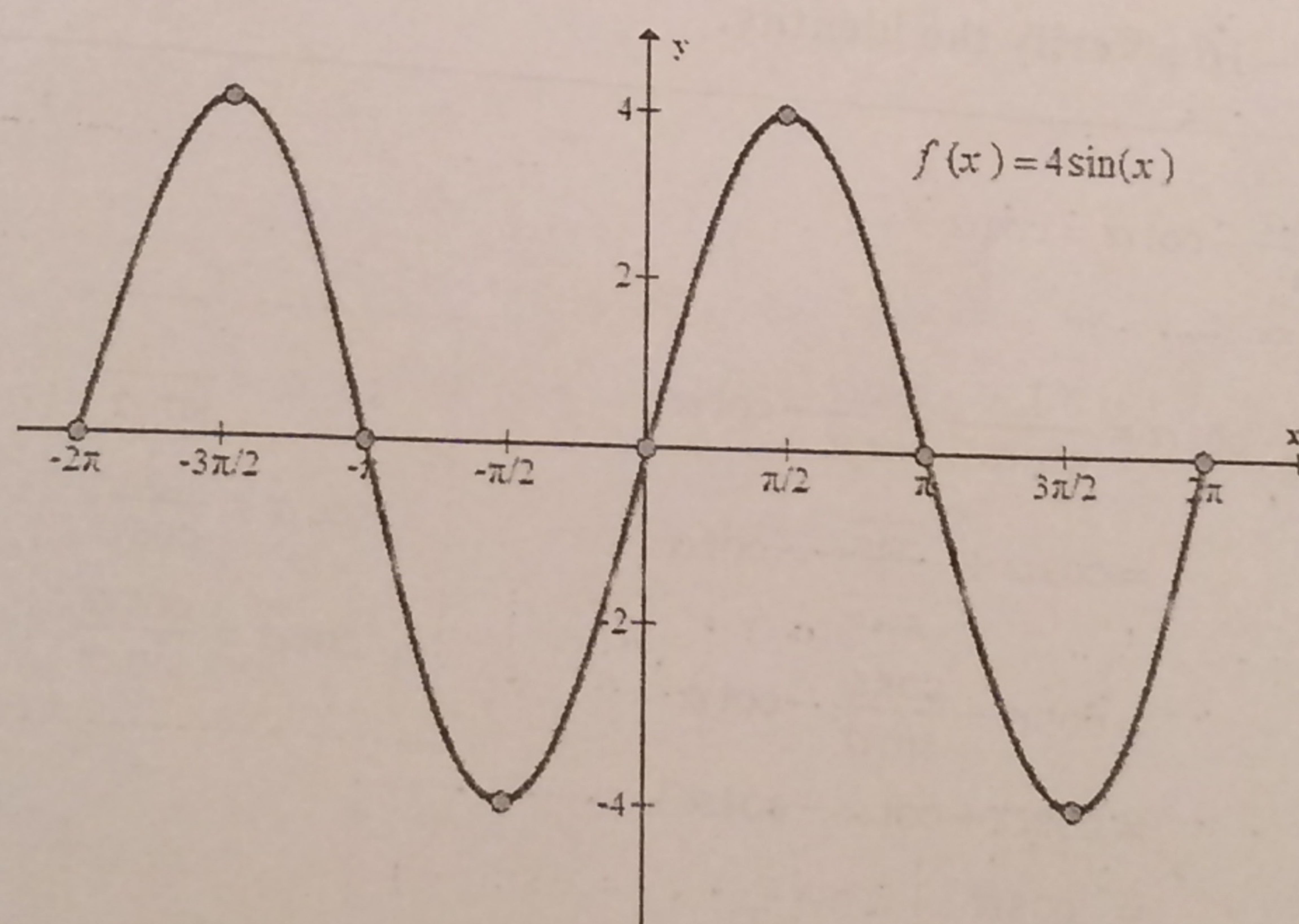
Exercise 9 ; Sketch the graph of f

a. $f(x) = 4\sin(x)$

solution:

نكون جدول يحتوي الزوايا الخاصة و نضع النقاط و توصيلها
كلما زادت عدد النقاط زادت دقة الرسم

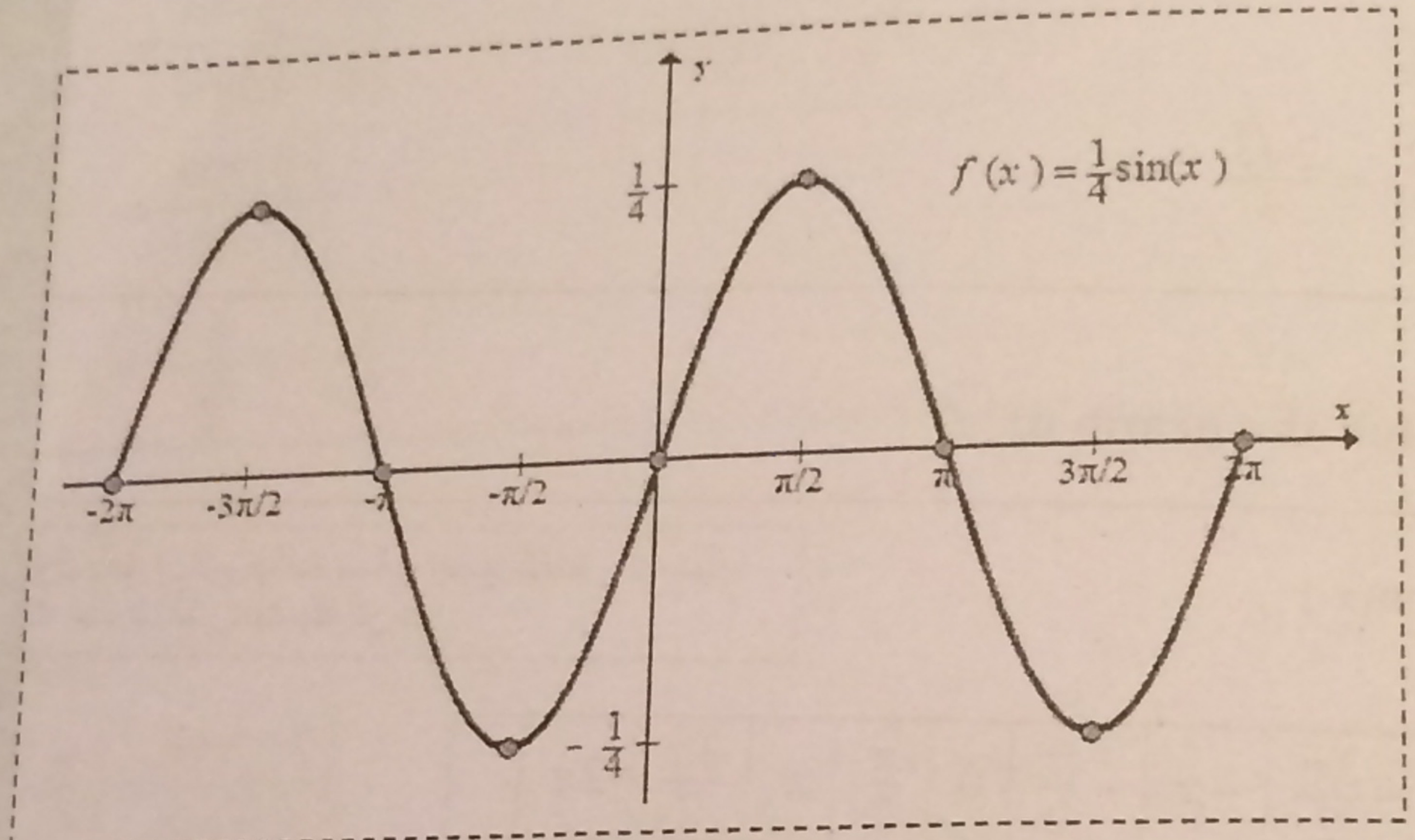
x	-2π	$-\frac{3\pi}{2}$	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	
$f(x)$	0	4	0	−4	0	4	0	−4	0	



b. $f(x) = \frac{1}{4}\sin(x)$

solution:

x	-2π	$-\frac{3\pi}{2}$	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	
$f(x)$	0	$\frac{1}{4}$	0	$-\frac{1}{4}$	0	$\frac{1}{4}$	0	$-\frac{1}{4}$	0	



Exercises 11 – 16 ; Verify the identity.

11. $\frac{1+\csc\alpha}{\sec\alpha} - \cot\alpha = \cos\alpha$

solution:

$$* \quad \frac{1+\csc\alpha}{\sec\alpha} - \cot\alpha = \frac{1}{\sec\alpha} + \frac{\csc\alpha}{\sec\alpha} - \cot\alpha$$

$$= \cos\alpha + \frac{\frac{1}{\sin\alpha}}{\frac{1}{\cos\alpha}} - \cot\alpha$$

$$= \cos\alpha + \frac{\cos\alpha}{\sin\alpha} - \cot\alpha$$

$$= \cos\alpha + \cot\alpha - \cot\alpha$$

$$= \cos\alpha$$

$$\csc\alpha = \frac{1}{\sin\alpha}$$

$$\sec\alpha = \frac{1}{\cos\alpha}$$

$$\cot\alpha = \frac{\cos\alpha}{\sin\alpha}$$

12. $2\sin^2(2t)+\cos(4t)=1$

solution:

نستخدم القاعدة

* $\sin^2(x)=\frac{1}{2}(1-\cos(2x))$
* $\sin^2(2t)=\frac{1}{2}(1-\cos(4t))$

* $$2\sin^2(2t)+\cos(4t)=2\cdot\frac{1}{2}\big(1-\cos(2(2t))\big)+\cos(4t)$$
$$=1-\cos(4t)+\cos(4t)$$
$$=1$$

14. $\dfrac{1}{\csc y-\cot y}=\csc y+\cot y$

solution:

* $$\frac{1}{\csc y-\cot y}=\frac{1}{\csc y-\cot y}\cdot\frac{\csc y+\cot y}{\csc y+\cot y}$$ بالضرب بالمرافق
$$=\frac{\csc y+\cot y}{\csc^2 y-\cot^2 y}$$
$$=\frac{\csc y+\cot y}{1}$$
$$=\csc y+\cot y$$

$\csc^2 x=\cot^2 x+1$

$\csc^2 x-\cot^2 x=1$

16. $\sin^4(2x)=\frac{3}{8}-\frac{1}{2}\cos(4x)+\frac{1}{8}\cos(8x)$

solution:

* $$\sin^4(2x)=\left(\sin^2(2x)\right)^2$$
نعوض عن $\sin^2(2x)=\frac{1}{2}(1-\cos 4x)$
$$=\left(\tfrac{1}{2}(1-\cos(4x))\right)^2$$
$$=\frac{1}{4}(1-\cos(4x))^2$$
نفك القوس التربيعي

$(A+B)^2=A^2+2AB+B^2$

$(1-\cos 4x)^2=1-2\cos 4x+\cos^2 4x$
$$=\frac{1}{4}\left(1-2\cos(2x)+\cos^2(4x)\right)$$
$$=\frac{1}{4}\left(1-2\cos(2x)+\tfrac{1}{2}(1+\cos(8x))\right)$$
نعوض عن $\cos^2(4x)=\frac{1}{2}(1+\cos 8x)$
$$=\frac{1}{4}\left(1-2\cos(2x)+\tfrac{1}{2}+\tfrac{1}{2}\cos(8x)\right)$$
$$=\frac{1}{4}\left(\tfrac{3}{2}-2\cos(2x)+\tfrac{1}{2}\cos(8x)\right)$$
$$=\frac{3}{8}-\frac{1}{2}\cos(2x)+\frac{1}{8}\cos(8x)$$

نظرية: اذا كان لدينا مستقيم l_1 و ميله m_1 . و مستقيم l_2 و ميله m_2 و كان المستقيمان متقاطعان و الزاوية بينهما θ

فان $\tan\theta = \dfrac{m_2 - m_1}{1 + m_1 m_2}$

مع ملاحظة أن m_2 هي ميل المستقيم ذو الزاوية الأكبر

ملاحظة هامة جدا : زاوية ميل المستقيم الأكبر هي m_2

- اذا كان m_1 و m_2 موجبتان فان m_2 هي الأكبر
- اذا كان m_1 و m_2 سالبتان فان m_2 هي الأكبر
- اذا كان m_1 و m_2 مختلفان الاشارة فان m_2 هي السالبة

Exercises 17 – 19 ; Find the tangent of the angle between the lines having the given slopes

17.

a. 1 and $\frac{1}{4}$

solution:

$m_2 = 1 \quad , \quad m_1 = \frac{1}{4}$

$$* \ \tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

اذا كان m_1 و m_2 موجبتان فان m_2 هي الأكبر

$$= \frac{1 - \frac{1}{4}}{1 + (1)\frac{1}{4}}$$

$$= \frac{\frac{3}{4}}{\frac{5}{4}} = \frac{3}{5}$$

b. 4 and $\frac{-5}{3}$

solution:

$m_2 = \frac{-5}{3}$, $m_1 = 4$

اذا كان m_2 و m_2 مختلفان الاشارة فان m_2 هي السالبة

* $\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$

$= \frac{\frac{-5}{3} - 4}{1 + 4 \cdot \frac{-5}{3}}$

$= \frac{-\frac{17}{3}}{-\frac{17}{3}} = 1$

19.

a. $-\frac{1}{2}$ and $\frac{2}{3}$

solution:

$m_2 = -\frac{1}{2}$, $m_1 = \frac{2}{3}$

* $\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$

$= \frac{\frac{-1}{2} - \frac{2}{3}}{1 + \frac{2}{3} \cdot \frac{-1}{2}}$

$= \frac{-\frac{7}{6}}{\frac{2}{3}} = -\frac{7}{4}$

b. $-\frac{5}{4}$ and $-\frac{7}{5}$

solution:

اذا كان m_1 و m_2 سالبتان فان m_2 هي الأكبر

$$-\frac{5}{4} > -\frac{7}{5}$$

$$m_2 = -\frac{5}{4} \quad , \quad m_1 = -\frac{7}{5}$$

$$* \quad \tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$= \frac{\frac{-5}{4} - \left(\frac{-7}{5}\right)}{1 + \frac{-7}{5} \cdot \frac{-5}{4}}$$

$$= \frac{\frac{3}{20}}{\frac{11}{4}} = \frac{3}{55}$$

20. Find the periods of the following functions

أفضل طريقة لايجاد دورة الدالة هي رسم الدالة و نوجد الفترة التي تتكرر فيها الدالة

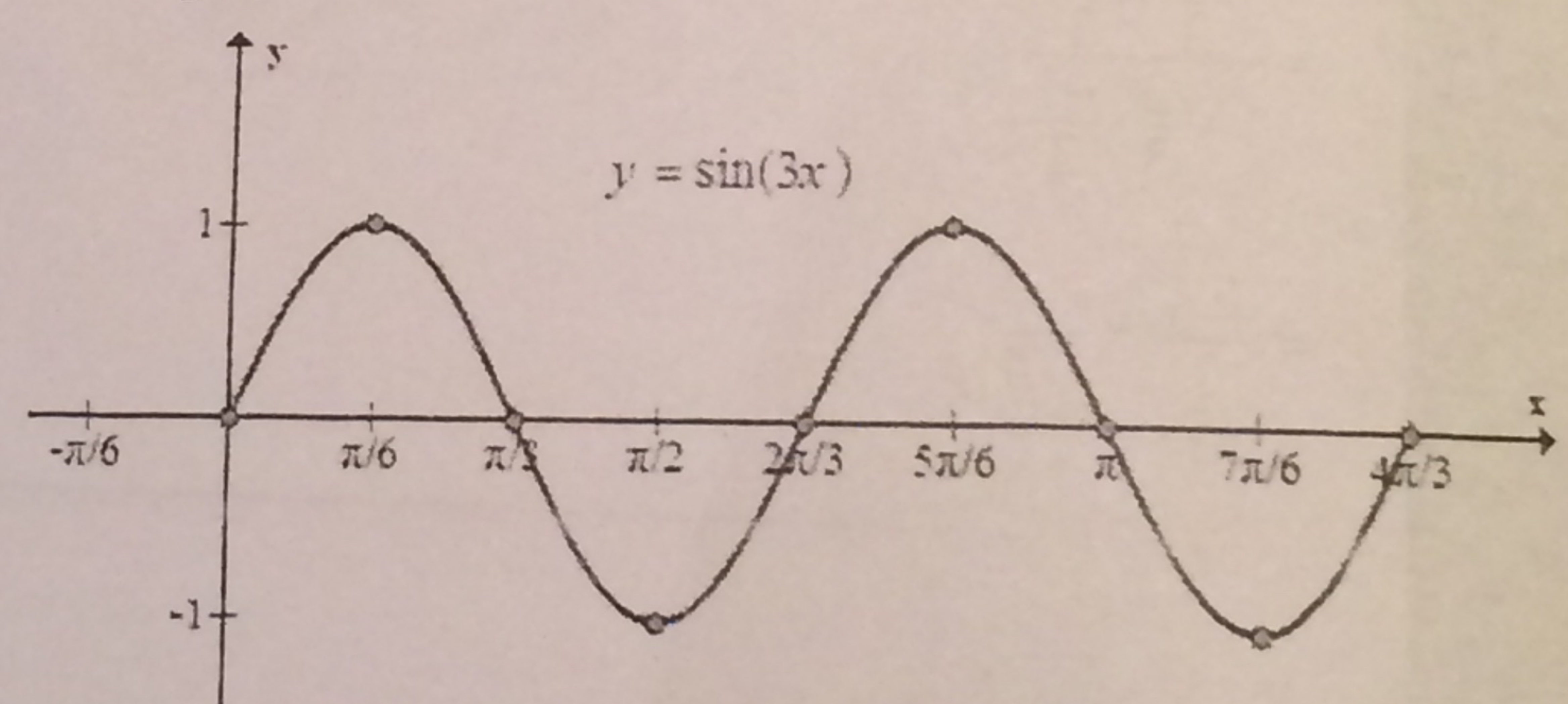
a. $f(x) = \sin(3x)$

solution:

The period of $\sin x$ is 2π

$$3x = 2\pi \Rightarrow x = \frac{2\pi}{3}$$

The period of $\sin(3x)$ is $\frac{2\pi}{3}$



b. $g(x) = 4\tan x$

solution:

the period of $\tan x$ is π

$\Rightarrow$ the period of $4\tan x$ is π

c. $h(x)=\cos\left(-2x+\frac{\pi}{2}\right)$

solution:

* $\cos\left(-2x+\frac{\pi}{2}\right)=-\sin(-2x)=+\sin(2x)$

the period of $\sin x$ is 2π

the peroid of $\sin(2x)$ is $2x=2\pi \Rightarrow x=\pi$

The period of $\cos\left(-2x+\frac{\pi}{2}\right)$ is π

d. $k(x)=|\sin x|$

solution:

the period of $\sin x$ is 2π

$\Rightarrow$ the period of $|\sin x|$ is π

e. $l(x)=\cot x$

solution:

The period of $\cot x$ is π